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Construction of Lorentz Invariant Amplitudes from Rest Frame Wave Functions in HQET – Application to Isgur-Wise Function

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Succeeding in predicting 0^+ and 1^+ states of D and D_s heavy mesons by our semi-relativistic quark potential model, we examine a method how to construct Lorentz-invariant scattering amplitudes and/or decay widths and develop a formulation to calculate Lorentz-boosted ones given the rest frame wave functions in our model.

To show how effective this is, we apply the formulation to calculate the semileptonic weak form factors out of the rest frame wave functions of heavy mesons and numerically calculate the dynamical $1/m_Q$ corrections to those for the process $\bar{B} \to D^{(*)}\ell\nu$ based on our model for heavy mesons. It is shown that nonvanishing expressions for $\rho_1(\omega) = \rho_2(\omega)$ and $\rho_3(\omega) = \rho_4(\omega) = 0$ are obtained in a special Lorentz frame, where $\rho_i(\omega)$ are the parameters used in the Neubert-Rieckert decomposition of form factors. Various values of form factors are estimated, which are compatible with recent experimental data as well as other theoretical calculations.

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I. INTRODUCTION

Discovery of narrow meson states $D_{s0}(2317)$ by BaBar [1] and $D_{s1}(2460)$ by CLEO [2] and the following confirmation by Belle [3, 5] has driven many theorists to explain these states since the former study of these states (Refs. [6], [7] and many more papers) using quark potential model seems to fail to reproduce these mass values. More recent experiments have found many other heavy-light mesons, broad $D_0^*(2308)$ and $D_1'(2427)$ mesons by Belle collaboration [8], which are identified as $c\bar{q}$ ($q = u/d$) excited ($\ell = 1$) bound states and have the same quantum numbers $j^P = 0^+$ and 1^+ as D_{sJ}, respectively (These mesons have quite different masses due to CLEO and FOCUS experiments and hence these are not yet definite. [4, 5]); narrow B and B_s states of $\ell = 1$, $B_1(5720)$, $B_2^*(5745)$, and $B_{s2}^*(5839)$ by CDF and D0 [9] whose decay widths are also narrow because of their decay through the D-waves; and seemingly radial excitations ($n = 2$) of 0^+ $D_{s0}(2860)$ state by BaBar [10] and 1^- $Ds^*(2715)$ state by Belle [11]. Furthermore $c\bar{c}$ quarkonium-like states have been discovered one after another, $X(3872)$, $X(3940)$, $Y(3940)$, $Z(3930)$, and $Y(4260)$.

These mesons triggered a series of study on spectroscopy of heavy-light and/or heavy-heavy hadrons using many kinds of ideas including our semirelativistic quark patential model. Since this paper is not intended to discuss these ideas, we just refer to the review articles [12, 13, 14]. In the former papers [15], we have formulated the method how to calculate spectrum of the heavy-light mesons to have the Shrödinger equation for a bound state, namely mass is expressed as an eigenvalue of Hamiltonian of the heavy-light system, in which a bound state consists of a heavy quark and a light antiquark and negative energy states of a heavy quark appear in the intermediate states in calculating an energy eigenvalue. To have more firm reliability to our formulation, we try to reproduce narrow B and B_s states of $\ell = 1$, $B_1(5720)$, $B_2^*(5745)$, and $B_{s2}^*(5839)$ together with higher states of J^P for D, D_s, B, and B_s [16], and also to reproduce radial excitations of 0^+ state of $D_{s0}(2860)$ and 1^- state of $Ds^*(2715)$ [17]. These calculations show our model's reliability since the above results fit well with experiments. What we need to do next is to show that our approach can also give a method to calculate scattering amplitudes and decay widths using the rest frame wave functions. We would like to concentrate on construction of such a formulation so let us leave the review paper [18] to present and explain other methods and their results since there are already many papers which give estimates for widths of many decay modes of heavy mesons.

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The heavy quark effective theory (HQET) has brought us a very convenient tool or idea to study heavy mesons/baryons including at least one heavy quark.[19, 20] Especially there have been many studies on the Isgur-Wise function [21]∼[30] but most of their calculations has used a simple-minded Gaussian form for the meson wave function or a solution to a single particle Dirac equation in a potential. Some other people used the Bethe-Salpeter solution for a meson wave function but it seems there has been no systematically developed way to calculate the higher order terms in $1/m_Q$ [25, 30] although some paper [31] has claimed so. Recently there appears a paper [32] in which the authors use the "relativistic" formulation to calculate masses of heavy mesons and apply it to calculate the Isgur-Wise functions for semileptonic B decays. Although this paper adopts a different approach from ours, they present how to calculate higher order corrections in $1/m_Q$ to form factors, which is one of the same purposes as we intend to do in this paper.

Most of the other calculations of form factors in the past used the so-called trace formalism developed by Falk et al. [19] which assumes the Lorentz covariance of the final form and fits it with the experimental data or the QCD sum rule [33] was used to obtain the relation with the quark/gluon condensates. In the former papers [15], we have developed a semi-relativistic formulation how to calculate the meson masses and wave functions for heavy-light $Q\bar{q}$ system with one heavy quark Q and one light antiquark $\bar{q}$, introducing phenomenological dynamics. That is, assuming Coulomb-like vector and confining scalar potentials to $Q\bar{q}$ bound states, we have expanded an effective Hamiltonian and then perturbatively solved Schrödinger equation in $1/m_Q$. Meson wave functions obtained thereby and expanded in $1/m_Q$ can be used in principle to calculate ordinary form factors or Isgur-Wise functions and their higher order corrections in $1/m_Q$ for semileptonic weak or other decay processes. However what we have obtained are wave functions in the rest frame so that we need to develop a method to obtain Lorentz invariant amplitudes or Lorentz-boosted wave functions. In this paper we would like to address the problem how accurately one can calculate scattering amplitudes and/or decay widths of heavy-light systems using the rest frame wave functions. In the following, explaining the results obtained in [15] we will show how to calculate the Lorentz invariant amplitudes taking the Isgur-Wise function as an example, which is general enough to apply our formulation to other scattering/decay amplitudes.

Following the former study [34] in which we derived the zeroth order form of the Isgur-Wise function in $1/m_Q$, we will develop a formulation how to calculate higher order corrections to the semi-leptonic weak form factors in $1/m_Q$ from our semi-relativistic wave functions for heavy mesons in the rest frame obtained in the former paper [15]. The problem how to construct a Lorentz-boosted wave function is in that there is ambiguity to determine space-time coordinates of two composite particles from information of one bound state, in this case a heavy meson. We will study four cases of reference frames for composite particles and then give a prescription how to calculate matrix elements of currents using the rest frame wave functions in Section II. In Section III, we will give form factors in the zeroth order for four reference frames, show that they agree with each other in the HQET limit, and give those in the first order for just one special reference frame. The lowest order calculation in $1/m_Q$ gives the numerical value of the slope for the Isgur-Wise function at the origin and the semi-leptonic weak form factors are calculated up to the first order in $1/m_Q$ in Section IV. It is derived that there are no dynamical contributions to the form factors, i.e., the first order corrections to the wave functions do not contribute to the form factors. In Section IV, studying $\bar{B} \to D\ell\nu$ and $\bar{B} \to D^*\ell\nu$ processes, physical quantities related to the CKM matrix elements are obtained. Finally Section V is devoted to summary and discussions obtained in this paper. All the details associated with calculation of form factors are given in Appendices.

II. FORMULATION

A. Schrödinger Equation

In the former papers [15], we have calculated mass spectrum of heavy mesons in a so-called Cornell potential model, which includes both scalar and vector potentials. Although we have needed only a stationary system, or rest frame wave function in [15], we must now treat a moving frame of a heavy meson to calculate weak form factors. Although we cannot work in a fully relativistic system as far as Hamiltonian formulation is concerned, we adopt the notations of the Nambu-Bethe-Salpeter equation as far as possible. First we define a heavy meson (X) wave function composed of a light anti-quark, $q^c(x)$, and a heavy quark, $Q(y)$, as

$$\langle 0|\, q^c_\alpha(t,\vec{x})\, Q_\beta(t,\vec{y})\, |X; P_X\,\{\ell\}\rangle = \psi^\ell_{X\,\alpha\,\beta}\left((0,\vec{x}-\vec{y})\,;P_X\right)\, e^{-iP_X\cdot y}, \tag{1}$$

where α and β denote Dirac indices of four-spinors, $P_{X\,\mu}$ heavy meson's four-momentum, $\{\ell\}$ a set of quantum numbers, and $P_X \cdot y = P_{X\,\mu}\, y^\mu$. Note that this definition of the wave function, ψ^ℓ_X, is minus sign times that of the former paper.[15] The heavy meson state is an eigen state of a four-momentum operator, $\mathcal{P}_\mu$,

$$\mathcal{P}_\mu\, |X; P_X\,\{\ell\}\rangle = P_{X\,\mu}\, |X; P_X\,\{\ell\}\rangle\,, \qquad P_{X\,0} = \sqrt{M_X^2 + \vec{P}_X^2}, \tag{2}$$

with M_X a heavy meson mass. In this case, the Schrödinger equation in a X moving frame is given by

$$H\,\psi_X^\ell\left((0,\vec{x}-\vec{y})\,;P_X\right)=\sqrt{M_X^2+\vec{P}_X^2}\,\psi_X^\ell\left((0,\vec{x}-\vec{y})\,;P_X\right), \tag{3}$$

where the Hamiltonian is given by

$$H=(\vec{\alpha}_q\cdot\vec{p}+\beta_q m_q)+\left[\vec{\alpha}_Q\cdot(\vec{P}_X-\vec{p})+\beta_Q m_Q\right]+\sum_{i,\,j}\beta_q O_{q\,i}V_{i\,j}(\vec{x}-\vec{y})\beta_Q O_{Q\,j}, \tag{4}$$

where $\vec{p}=-i\vec{\nabla}_x$ and $\vec{P}_X$ is a heavy meson momentum. In our case, Eq. (6) gives $O_{q\,i}=O_{Q\,i}=1$ for a scalar potential, $V_{i\,j}(\vec{x}-\vec{y})=S(r)$ and $O_{q\,i}=\gamma_{q\,\mu}$ and $O_{Q\,j}=\gamma_Q{}^\mu$ for a vector potential, $V_{i\,j}(\vec{x}-\vec{y})=V(r)$ with $r=|\vec{x}-\vec{y}|$. Actually we have included a transverse part of vector potential and the total potential in Eq. (4) is given by

$$\beta_q\beta_Q\,S(r)+\left\{1-\frac{1}{2}\left[\,\vec{\alpha}_q\cdot\vec{\alpha}_Q+(\vec{\alpha}_q\cdot\vec{n})\,(\vec{\alpha}_Q\cdot\vec{n})\,\right]\right\}V(r), \tag{5}$$

with

$$S(r)=\frac{r}{a^2}+b,\qquad V(r)=-\frac{4}{3}\frac{\alpha_s}{r},\qquad \vec{n}=\frac{\vec{r}}{r}. \tag{6}$$

What we have done in the former paper [15] is to solve the Schrödinger equation given by Eq. (3) with $\vec{P}_X=\vec{0}$ order by order in $1/m_Q$ by expanding H, ψ_X^ℓ, and M_X in $1/m_Q$, to numerically calculate the mass spectrum, i.e., an eigenvalue, M_X, in each order of perturbation, and then to relate the results with those of HQET.

B. Lorentz Boost

In order to calculate the decay rate we need to Lorentz-boost the rest frame wave function which we have obtained in the former papers. To explain the idea we take $\bar{B}\to D^{(*)}\ell\nu$ decay process as an example and express the wave function in a moving frame in terms of the one in a rest frame.

Assume that the heavy meson is moving in the $+z$ direction with a velocity β. Then we consider two cases below of the composite particles, i.e., q^c and Q. In the following for the initial state of a heavy meson the notations, $q^c(x^0,\vec{x})$ and $Q(y^0,\vec{y})$, are adopted and for the final state $q^c(x'^0,\vec{x}\,')$ and $Q(y'^0,\vec{y}\,')$. A relation between the constituent particles in the rest and moving frames is given by

$$\mathcal{G}\,q^c_\alpha(x^0,\vec{x})\,\mathcal{G}^{-1}\;=\;G^{-1}_{\alpha\,\beta}\,q^c_\beta(x'^0,\vec{x}\,'), \tag{7}$$

$$\mathcal{G}\,Q_\alpha(y^0,\vec{y})\,\mathcal{G}^{-1}\;=\;G^{-1}_{\alpha\,\beta}\,Q_\beta(y'^0,\vec{y}\,'), \tag{8}$$

where $\mathcal{G}$ is a Lorentz-boost operator, G is its spinor representation, and their definitions are given by

$$\mathcal{G}\,\Big|X;\Big(M_X,\vec{0}\Big)\,\{\ell\}\Big\rangle=|X;P_X\,\{\ell\}\rangle \tag{9}$$

$$G=\cosh\frac{\varphi}{2}+\alpha^3\sinh\frac{\varphi}{2},\qquad \beta=\tanh\varphi,\qquad \gamma=\frac{1}{\sqrt{1-\beta^2}}=\cosh\varphi. \tag{10}$$

We have assumed that a heavy meson is boosted in $+z$ direction. Here $\vec{\alpha}=\gamma^0\vec{\gamma}$, β and γ being a velocity and a Lorentz factor of a heavy meson, respectively, which can be expressed in terms of one parameter, φ.

Even if two constituent quarks have equal time in one frame, they have different times in another frame. In order to pull back the time of the heavy quark to the one of the light quark, we make an approximation that the heavy quark propagates freely in a short time interval, that is,

$$Q_\beta\left(t+\delta t,\vec{y}\right)\simeq\exp\left(-im_Q\gamma\delta t\right)Q_\beta\left(t,\vec{y}\right). \tag{11}$$

Below we consider two cases. In the first case two constituent quarks have equal time in the rest frame of the meson, and we apply the above approximation to the heavy quark in the moving frame. In the second case two constituents have equal time in the moving frame of the meson, and we apply the approximation to the heavy quark in the rest frame. In two cases different relations are derived between the wave function of the meson with finite momentum and the one of the meson at rest.

i) $t = x^0 = y^0$ and $x'^0 \neq y'^0$ (equal time in the rest frame)

$$\begin{aligned} x'^3 &= x^3 \cosh\varphi + t \sinh\varphi, \quad & x'^0 &= t\cosh\varphi + x^3 \sinh\varphi, \\ y'^3 &= y^3 \cosh\varphi + t \sinh\varphi, \quad & y'^0 &= t\cosh\varphi + y^3 \sinh\varphi. \end{aligned} \tag{12}$$

In this case an approximate relation between the rest and moving frame wave functions is given by

$$\psi^{\ell}_{X\,\alpha\,\beta}\left((0,\vec{x})\,;P_X\right) \simeq G_{\alpha\,\gamma} G_{\beta\,\delta}\, \psi^{\ell}_{X\,\gamma\,\delta}\left(\left(0,\vec{x}_\perp,\gamma^{-1}x^3\right);\left(M_X,\vec{0}\right)\right)\, e^{i(M_X - m_Q)\gamma\,\beta\,x^3}. \tag{13}$$

On the other hand

ii) $t' = x'^0 = y'^0$ and $x^0 \neq y^0$ (equal time in the moving frame)

$$\begin{aligned} x^3 &= x'^3 \cosh\varphi - t' \sinh\varphi, \quad & x^0 &= t'\cosh\varphi - x'^3 \sinh\varphi, \\ y^3 &= y'^3 \cosh\varphi - t' \sinh\varphi, \quad & y^0 &= t'\cosh\varphi - y'^3 \sinh\varphi. \end{aligned} \tag{14}$$

In this case an approximate relation between the rest and moving frame wave functions is given by

$$\psi^{\ell}_{X\,\alpha\,\beta}\left((0,\vec{x})\,;P_X\right) \simeq G_{\alpha\,\gamma} G_{\beta\,\delta}\, \psi^{\ell}_{X\,\gamma\,\delta}\left(\left(0,\vec{x}_\perp,\gamma x^3\right);\left(M_X,\vec{0}\right)\right)\, e^{i(M_X - m_Q)\gamma\,\beta\,x^3}. \tag{15}$$

Derivation of Lorentz boosted wave functions given by Eqs. (13, 15) is given in Appendix B.

C. Wave Function

The explicit form of the wave function we obtained in the former paper has a form,

$$U_X(p)\psi^{\ell}_X\,(\vec{x}) = \psi^{\ell}_{X\ \mathrm{FWT}}(\vec{x}), \tag{16a}$$

$$U_X(p) \equiv U_c U_{\mathrm{FWT}}(p), \qquad \psi^{\ell}_X\,(\vec{x}) \equiv \psi^{\ell}_X\left((0,\vec{x})\,;\left(M_X,\vec{0}\right)\right), \tag{16b}$$

$$\psi^{\ell}_{X\ \mathrm{FWT}}(\vec{x}) = \psi^{\ell}_{X\,0}(\vec{x}) + \psi^{\ell}_{X\,1}(\vec{x}) + \ldots, \tag{16c}$$

$$\psi^{\ell}_{X\,0}(\vec{x}) = \Psi^{+}_{\ell}(\vec{x}) \equiv \sqrt{M_X}\,\begin{pmatrix} 0 & \Psi^{k}_{j\,m} \end{pmatrix}, \tag{16d}$$

We have solved the Schrödinger equation in terms of the wave function $\psi^{\ell}_{X\ \mathrm{FWT}}$ not ψ^{ℓ}_X. Here the 4×4 wave function, ψ^{ℓ}_X, is transformed just once with the Foldy-Wouthuysen-Tani transformation, U_{FWT}, acting on a heavy quark and with the charge conjugation operator, U_c, into $\psi^{\ell}_{X\ \mathrm{FWT}}$, and its i-th order expanded in $1/m_Q$ is given by $\psi^{\ell}_{X\,i}$. Higher order corrections depend on the positive and negative components of a heavy quark, $\Psi^{\pm}_{\ell}$, which are given by

$$\Psi^{+}_{\ell}(\vec{x}) \equiv \sqrt{M_X}\,\begin{pmatrix} 0 & \Psi^{k}_{j\,m}(\vec{x}) \end{pmatrix}, \qquad \Psi^{-}_{\ell}(\vec{x}) \equiv \sqrt{M_X}\,\begin{pmatrix} \Psi^{k}_{j\,m}(\vec{x}) & 0 \end{pmatrix}. \tag{17}$$

Here $\vec{p} = \vec{p}_Q$ appearing in the argument of $U_X(p)$ above is an initial momentum of a heavy quark, and henceforth the color index ($N_c = 3$) is omitted since the form of the wave function is the same for all colors, ℓ stands for a set of quantum numbers, j, m, and k, and the positive/negative component wave functions, $\Psi^{\pm}_{\ell}$, are given in terms of a 4×2 wave function, $\Psi^{k}_{j\,m}(\vec{x})$, as

$$\Psi^{k}_{j\,m}(\vec{x}) = \frac{1}{r}\begin{pmatrix} u_k(r) \\ -i\,v_k(r)\,(\vec{n}\cdot\vec{\sigma}) \end{pmatrix} y^{k}_{j\,m}(\Omega), \tag{18}$$

where $r = |\vec{x}|$, $\vec{n} = \vec{x}/r$, j is a total angular momentum of a meson, m is its z component, k is a quantum number which takes only values, $k = \pm j$, $\pm(j+1)$ and $\neq 0$. More details are given in Appendix C.

We have shown in the former paper that $\Psi^{+}_{\ell}(\vec{x})$ is an eigenstate of the operator, $K \equiv -\beta_q(\vec{\Sigma}_q\cdot\vec{\ell}+1)$ with an eigenvalue k and we have classified the spectra in terms of k. People normally classify the spectra in terms of ${s_\ell}^P$ which is defined to be an eigen value $s_\ell\,(s_\ell+1)$ of the operator $(\vec{s}_\ell)^2$ [35] together with parity, P. One can show the following direct relation among k, s_ℓ, and P, i.e., s_ℓ and P can be explicitly written only in terms of k as, [36]

$$k \;=\; \pm\left(s_\ell + \frac{1}{2}\right) \quad \text{or} \quad s_\ell = |k| - \frac{1}{2}. \tag{19a}$$

$$P \;=\; \frac{k}{|k|}(-1)^{|k|+1} \tag{19b}$$

where defined are

$$\vec{s}_\ell \equiv \vec{\ell} + \frac{1}{2}\vec{\Sigma}_q, \quad (\vec{s}_\ell)^2 = \vec{\ell}^{\,2} + \vec{\ell}\cdot\Sigma_q + \frac{3}{4}, \quad \vec{\ell} = -i\vec{r}\times\vec{\nabla}, \quad \vec{\Sigma}_q = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix}. \tag{20}$$

The corresponding physical quantities in general between k and s_ℓ and the first few states are listed in Tables I and II, respectively.

Looking at Tables I and II, there are both advantages and disadvantages for using these quantities to classify the heavy mesons. s_ℓ^P has a relatively intuitive meaning in itself that it expresses the total angular momentum of the light degrees of freedom while k does not. However, k includes an information of parity while s_ℓ does not hence we need to add P to denote it as s_ℓ^P. One can certainly conclude that either one can classify the heavy meson spectra equally well, i.e., they are equivalent to each other.

D. Normalization

Normalization of the state, $|X; P_X\,\{\ell\}\rangle$, is formally given by

$$\langle X; P'_X\,\{\ell'\}\,|X; P_X\,\{\ell\}\rangle = (2\pi)^3\, 2P_{X0}\,\delta^3\left(\vec{P}_X - \vec{P}'_X\right)\delta_{\ell',\ell}, \tag{21}$$

or

$$\int d^3z\,\mathrm{tr}\left[\psi_X^{\ell'\,\dagger}\left((0,\vec{z})\,;P_X\right)\psi_X^{\ell}\left((0,\vec{z})\,;P_X\right)\right] \simeq 2P_{X0}\,\delta_{\ell\,\ell'}, \tag{22}$$

whose derivation is given in Appendix B. Actually the normalization given by Eq. (22) does not hold because of approximations adopted in the former subsection, and hence instead of Eq. (22) we will define the normalization in the rest frame as follows,

$$\int d^3z\,\mathrm{tr}\left[\psi_X^{\ell'\,\dagger}\left((0,\vec{z})\,;\left(M_X,\vec{0}\right)\right)\psi_X^{\ell}\left((0,\vec{z})\,;\left(M_X,\vec{0}\right)\right)\right] = 2M_X\,\delta_{\ell\,\ell'}. \tag{23}$$

In order to calculate the normalization in a moving frame we have to specify the quantum numbers we are interested in. Since we would like to compute the Isgur-Wise function and its higher orders, the cases of the pseudoscalar state 0^- and vector state 1^- are calculated in the following.

Normalization condition given by Eq. (23) can be rewritten in terms of $\psi_{X\,\mathrm{FWT}}^{\ell}(\vec{x})$ as,

$$\int d^3z\,\mathrm{tr}\left[{\psi_{X\,\mathrm{FWT}}^{\ell'}}^{\dagger}(\vec{x})\,\psi_{X\,\mathrm{FWT}}^{\ell}(\vec{x})\right] = 2M_X\,\delta_{\ell\,\ell'}. \tag{24}$$

All the details on the wave functions are given in Appendix C.

In order to calculate normalization of the wave function in a moving frame and/or matrix elements of the semi-leptonic decay processes, we need to develop a formulation of their calculations in terms of the rest frame wave functions. Let us assume that the physical quantity is already given in terms of the rest frame wave functions and rewrite them in terms of the FWT-transformed ones as

$$\begin{aligned}
&\int d^3x\,\left(\mathcal{O}_q\right)_{\alpha\,\alpha'}\left(\mathcal{O}_Q\right)_{\beta\,\beta'}A'^{\,*}_{\alpha\,\gamma}B'^{\,*}_{\beta\,\delta}\psi^{\ell'\,*}_{X'\,\gamma\,\delta}A_{\alpha'\,\gamma'}B_{\beta\,\delta'}\psi^{\ell}_{X\,\gamma'\,\delta'} = \int d^3x\,\mathrm{tr}\left(\psi^{\ell'\,\dagger}_{X'}A'^{\,\dagger}\mathcal{O}_qA\psi^{\ell}_XB^{\mathrm{T}}\mathcal{O}_Q^{\mathrm{T}}B'^{\,*}\right) \\
&= \int d^3x\,\mathrm{tr}\left[\psi^{\ell'\,\dagger}_{X'\,\mathrm{FWT}}A'^{\,\dagger}\mathcal{O}_qA\psi^{\ell}_{X\,\mathrm{FWT}}U_X^{-1}(p_Q)B^{\mathrm{T}}\mathcal{O}_Q^{\mathrm{T}}B'^{\,*}U_{X'}(p'_Q)\right],
\end{aligned} \tag{25}$$

where $(\mathcal{O}_q)_{\alpha\,\alpha'}$, $A'_{\alpha\,\gamma}$, and $A_{\alpha\,\gamma'}$ act on light quarks while $(\mathcal{O}_Q)_{\beta\,\beta'}$, $B'_{\beta\,\delta}$ and $B_{\beta\,\delta'}$ on heavy quarks, and use has been made of,

$$\begin{aligned}
&\psi^{\ell}_X = U_X^{-1}(p_Q)\otimes\psi^{\ell}_{X\,\mathrm{FWT}}(p_Q) = \psi^{\ell}_{X\,\mathrm{FWT}}\,U_X^{-1}(p_Q), \qquad \psi^{\ell}_{X'} = \psi^{\ell}_{X\,'\,\mathrm{FWT}}\,U_X^{-1}(p'_Q), \\
&U_X^{-1}(p_Q)B^{\mathrm{T}}\mathcal{O}_Q^{\mathrm{T}}B'^{\,*}U_{X'}(p'_Q) = U^{-1}_{X\,\mathrm{FWT}}(p_Q)U_c^{-1}B^{\mathrm{T}}\mathcal{O}_Q^{\mathrm{T}}B'^{\,*}U_cU_{X\,'\,\mathrm{FWT}}(p'_Q).
\end{aligned} \tag{26}$$

In the course of obtaining relativistic results out of the rest frame wave functions, we calculate normalization of the wave function in a moving frame in two extreme cases, i) $t = x^0 = y^0$ and ii) $t' = x'^0 = y'^0$. The FWT-transformed rest frame wave function is given by, up to the first order in $1/m_Q$, [15]

$$\psi^{\ell}_{X\,\mathrm{FWT}}(0^-) \;=\; \Psi^{+}_{-1} + c^{-1,\,1}_{1^-}\Psi^{-}_{1} + O\left(1/m_Q^2\right), \tag{27a}$$

$$\psi^{\ell}_{X\,\mathrm{FWT}}(1^-) \;=\; \Psi^{+}_{-1} + c^{-1,\,2}_{1^+}\Psi^{+}_{2} + c^{-1,\,1}_{1^-}\Psi^{-}_{1} + c^{-1,\,-2}_{1^-}\Psi^{-}_{-2} + O\left(1/m_Q^2\right), \tag{27b}$$

where only the value of a k quantum number is written as a subscript of $\Psi_\ell^\pm$. The total angular momentum, j, though not given explicitly, should be the same on both sides of equations and the coefficients, $c_{1\pm}^{k,k'}$, are given in [15] whose explicit expressions are not necessarily known here since we shall show that the higher order corrections do not contribute to the physical quantities in this order, $O(1/m_Q)$. Actually Eqs. (27) can be derived from conservation of the total angular momentum and parity alone without explicit interaction terms specified since we know the complete set of eigen functions with j, m, and k quantum numbers. Details are given in Appendix C.

Now we calculate normalization up to the first order in $1/m_Q$ below.

i) $t = x^0 = y^0$ and $x'^0 \neq y'^0$ (equal time at the rest frame)

Using Eqs. (13, 24, 25) one obtains, up to the first order in $1/m_Q$ and both for 0^- and 1^- states given by Eqs. (27),

$$\int d^3x \,\mathrm{tr}\left[\psi_X^{\ell'\,\dagger}\left((0,\vec{x})\,;P_X\right)\psi_X^{\ell}\left((0,\vec{x})\,;P_X\right)\right] = 2M_X\gamma^3 + O\left(1/m_Q^2\right). \tag{28}$$

ii) $t' = x'^0 = y'^0$ and $x^0 \neq y^0$ (equal time at the moving frame)

Similarly using Eq. (15, 24, 25) one obtains,

$$\int d^3x \,\mathrm{tr}\left[\psi_X^{\ell'\,\dagger}\left((0,\vec{x})\,;P_X\right)\psi_X^{\ell}\left((0,\vec{x})\,;P_X\right)\right] = 2M_X\gamma + O\left(1/m_Q^2\right). \tag{29}$$

Only this case agrees with the relativistic normalization given by Eq. (22).

E. Matrix Elements

The light anti-quark is treated as a spectator. That is, it does not interact with other particles except for gluons represented by potentials in our model. The heavy quark has a weak vertex and its current is in general given by

$$j(t,\vec{x}) = Q^\dagger_{X'\,\alpha}(t,\vec{x})\,\mathcal{O}_{\alpha\,\beta}\,Q_{X\,\beta}(t,\vec{x}), \tag{30}$$

where $\mathcal{O}$ is some 4×4 matrix. By inserting the number operator of the anti-quark and by assuming the vacuum dominance among intermediate states, we obtain the formula for the matrix element of the above current, Eq. (30), between heavy mesons as follows.

$$\begin{aligned}&\langle X'; P_{X'}\,\{\ell'\}\,|j(t,\vec{x})|\,X; P_X\,\{\ell\}\rangle \\ &\quad\simeq \int d^3y\,\mathrm{tr}\left[\psi_{X'}^{\ell'\,\dagger}\left((0,\vec{y}-\vec{x})\,;P_{X'}\right)\left(\mathcal{O}\otimes\psi_X^{\ell}\left((0,\vec{y}-\vec{x})\,;P_X\right)\right)\right]\,e^{-i(P_X-P_{X'})\cdot x},\end{aligned} \tag{31}$$

or

$$\left\langle X'; P_{X'}\,\{\ell'\}\left|j(0,\vec{0})\right|X; P_X\,\{\ell\}\right\rangle \simeq \int d^3x\,\mathrm{tr}\left[\psi_{X'}^{\ell'\,\dagger}\left((0,\vec{x})\,;P_{X'}\right)\,\psi_X^{\ell}\left((0,\vec{x})\,;P_X\right)\,\mathcal{O}^{\mathrm{T}}\right]. \tag{32}$$

Now we would like to evaluate the matrix element of the process, $\bar{B}\to D^{(*)}\ell\nu_\ell$, in two frames, i.e., $\bar{B}$ rest frame and the Breit frame. (The velocities of $\bar{B}$ and D have the same magnitude and have the opposite directions.) In each frame we apply two approximate relations Eq. (12) and Eq. (14) As has been mentioned in the former subsection the normalization condition Eq. (22) of the wave function does hold in the approximation ii) (equal time in the moving frame) but does not hold in the approximation i) (equal time in the rest frame). In the approximation i) we need to renormalize the matrix elements by the normalization of the wave function and we redefine the matrix elements of the current as follows:

$$\left\langle D^{(*)}\left|j(0,\vec{0})\right|\bar{B}\right\rangle = \frac{\sqrt{2P_{\bar{B}\,0}2P_{D^{(*)}\,0}}\int d^3x\,\mathrm{tr}\left[\psi_{D^{(*)}}^{\ell'\,\dagger}\left((0,\vec{x})\,;P_{D^{(*)}}\right)\,\psi_{\bar{B}}^{\ell}((0,\vec{x})\,;P_{\bar{B}})\mathcal{O}^{\mathrm{T}}\right]}{\sqrt{\int d^3x\,\mathrm{tr}\left|\psi_{D^{(*)}}^{\ell'}((0,\vec{x})\,;P_{D^{(*)}})\right|^2\int d^3x\,\mathrm{tr}\left|\psi_{\bar{B}}^{\ell}((0,\vec{x})\,;P_{\bar{B}})\right|^2}}, \tag{33}$$

whose the numerators are calculated below using the results of the former subsection II B, while the denominators are calculated in the former subsection II D and given by Eqs. (28, 29).

1) $\bar{B}$ rest frame
1-i) $t = x^0 = y^0$ for $D^{(*)}$ meson

In this case the matrix element is given by, within our approximations,

$$\begin{aligned}
&\int d^3x \,\mathrm{tr}\left[\psi^{\ell'\,\dagger}_{D^{(*)}}\left((0,\vec{x})\,;P_{D^{(*)}}\right)\,\psi^{\ell}_{\bar{B}}((0,\vec{x})\,;P_{\bar{B}})\mathcal{O}^{\mathrm{T}}\right] \\
&= \int d^3x \, G^*_{\gamma\,\delta} G^*_{\alpha\,\epsilon}\,\psi^{\ell'}_{D^{(*)}}{}^{*}_{\delta\,\epsilon}\left(\left(0,\vec{x}_\perp,\gamma^{-1}x^3\right);\left(M_{D^{(*)}},\vec{0}\right)\right)\mathcal{O}_{\alpha\,\beta}\,\psi^{\ell}_{\bar{B}\,\gamma\,\beta}\left(\vec{x}\right)e^{-i\left(M_{D^{(*)}}-m_c\right)\gamma\,\beta\,x^3},
\end{aligned} \tag{34}$$

1-ii) $t' = x'^0 = y'^0$ for $D^{(*)}$ meson

In this case the matrix element is given by,

$$\begin{aligned}
&\int d^3x \,\mathrm{tr}\left[\psi^{\ell'\,\dagger}_{D^{(*)}}\left((0,\vec{x})\,;P_{D^{(*)}}\right)\,\psi^{\ell}_{\bar{B}}((0,\vec{x})\,;P_{\bar{B}})\mathcal{O}^{\mathrm{T}}\right] \\
&= \int d^3x \, G^*_{\gamma\,\delta} G^*_{\alpha\,\epsilon}\,\psi^{\ell'}_{D^{(*)}}{}^{*}_{\delta\,\epsilon}\left(\left(0,\vec{x}_\perp,\gamma x^3\right);\left(M_{D^{(*)}},\vec{0}\right)\right)\mathcal{O}_{\alpha\,\beta}\,\psi^{\ell}_{\bar{B}\,\gamma\,\beta}\left(\vec{x}\right)e^{-i\left(M_{D^{(*)}}-m_c\right)\gamma\,\beta\,x^3},
\end{aligned} \tag{35}$$

2) Breit frame
2-i) $t = x^0 = y^0$ for $\bar{B}$ and $D^{(*)}$ mesons

In this case the matrix element is given by, within our approximations,

$$\begin{aligned}
&\int d^3x \,\mathrm{tr}\left[\psi^{\ell'\,\dagger}_{D^{(*)}}\left((0,\vec{x})\,;P_{D^{(*)}}\right)\,\psi^{\ell}_{\bar{B}}((0,\vec{x})\,;P_{\bar{B}})\mathcal{O}^{\mathrm{T}}\right] \\
&= \gamma \int d^3x \,\psi^{\ell'}_{D^{(*)}}{}^{*}_{\gamma\,\alpha}\left(\vec{x}\right)\left(G^\dagger \mathcal{O} G^{-1}\right)_{\alpha\,\beta}\psi^{\ell}_{\bar{B}\,\gamma\,\beta}\left(\vec{x}\right)e^{-\frac{i}{2}\left(M_{D^{(*)}}-m_c+M_B-m_b\right)\gamma^2\,\beta\,x^3}.
\end{aligned} \tag{36}$$

2-ii) $t' = x'^0 = y'^0$ for $\bar{B}$ and $D^{(*)}$ mesons

In this case the matrix element is given by,

$$\begin{aligned}
&\int d^3x \,\mathrm{tr}\left[\psi^{\ell'\,\dagger}_{D^{(*)}}\left((0,\vec{x})\,;P_{D^{(*)}}\right)\,\psi^{\ell}_{\bar{B}}((0,\vec{x})\,;P_{\bar{B}})\mathcal{O}^{\mathrm{T}}\right] \\
&= \gamma^{-1} \int d^3x \,\psi^{\ell'}_{D^{(*)}}{}^{*}_{\gamma\,\alpha}\left(\vec{x}\right)\left(G^\dagger \mathcal{O} G^{-1}\right)_{\alpha\,\beta}\psi^{\ell}_{\bar{B}\,\gamma\,\beta}\left(\vec{x}\right)e^{-\frac{i}{2}\left(M_{D^{(*)}}-m_c+M_B-m_b\right)\beta\,x^3}.
\end{aligned} \tag{37}$$

III. SEMI-LEPTONIC WEAK FORM FACTOR

Using the results derived in the former section we can now calculate semi-leptonic weak form factors including the Isgur-Wise function. In this case the currents are given by

$$j_\mu = c^\dagger\left(t,\vec{x}\right)\gamma^0\gamma_\mu\, b\left(t,\vec{x}\right) \tag{38a}$$

$$j^5_\mu = c^\dagger\left(t,\vec{x}\right)\gamma^0\gamma_\mu\gamma_5\, b\left(t,\vec{x}\right) \tag{38b}$$

Define the six independent semi-leptonic weak form factors as [21]

$$\langle D|\, j_\mu(0,\vec{0})\left|\bar{B}\right\rangle = \sqrt{M_B M_D}\left(\xi_+(\omega)(v_{\bar{B}}+v_D)_\mu + \xi_-(\omega)(v_{\bar{B}}-v_D)_\mu\right), \tag{39a}$$

$$\langle D^*|\, j_\mu(0,\vec{0})\left|\bar{B}\right\rangle = i\,\sqrt{M_B M_{D^*}}\xi_V(\omega)\,\epsilon_{\mu\nu\rho\sigma}\epsilon^{*\,\nu}v^\rho_{D^*}\,v^\sigma_{\bar{B}}, \tag{39b}$$

$$\begin{aligned}
\langle D^*|\, j^5_\mu(0,\vec{0})\left|\bar{B}\right\rangle &= \sqrt{M_B M_{D^*}}\left((1+\omega)\xi_{A_1}(\omega)\,\epsilon^*_\mu\right. \\
&\quad \left.-\xi_{A_2}(\omega)\,\left(\epsilon^*\cdot v_{\bar{B}}\right)v_{\bar{B}\,\mu} - \xi_{A_3}(\omega)\,\left(\epsilon^*\cdot v_{\bar{B}}\right)v_{D^*\,\mu}\right),
\end{aligned} \tag{39c}$$

with

$$v^\mu_X = P^\mu_X/M_X, \quad \text{for } X = \bar{B},\ D^{(*)}, \qquad \omega = v_{\bar{B}}\cdot v_D. \tag{40}$$

A. Zeroth Order (Isgur-Wise Function)

After some calculations, all the above form factors are proportional to the Isgur-Wise function, $\xi(\omega)$, or vanish and are given by

$$\xi_+(\omega) = \xi_V(\omega) = \xi_{A_1}(\omega) = \xi_{A_3}(\omega) = \xi(\omega), \tag{41a}$$
$$\xi_-(\omega) = \xi_{A_2}(\omega) = 0, \tag{41b}$$

Below we will show briefly how to only calculate $\xi_+(\omega) = \xi(\omega)$ from Eq. (39a) since the rests are similarly obtained. All the details are given in the Appendix E. Define the Isgur-Wise function as

$$\begin{aligned}\left\langle D \left| j_0(0,\vec{0}) \right| \bar{B} \right\rangle &= \frac{\sqrt{2P_{\bar{B}\,0}2P_{D\,0}}\int d^3x \,\mathrm{tr}\left[\psi^{\ell'\,\dagger}_{D\,0}((0,\vec{x})\,;P_D)\;\psi^{\ell}_{\bar{B}\,0}((0,\vec{x})\,;P_{\bar{B}})\right]}{\sqrt{\int d^3x\,\mathrm{tr}\left|\psi^{\ell'}_{D\,0}((0,\vec{x})\,;P_D)\right|^2\int d^3x\,\mathrm{tr}\left|\psi^{\ell}_{\bar{B}\,0}((0,\vec{x})\,;P_{\bar{B}})\right|^2}} \\ &= \sqrt{M_B M_D}\xi(\omega)(v_{\bar{B}} + v_D)_0. \end{aligned}\tag{42}$$

The expression, Eq. (42), can be further reduced in two cases of $\bar{B}$ frames as follows.

1) $\bar{B}$ rest frame

In this frame $\bar{B}$ is at rest, D is moving in the $+z$ direction with velocity β and $\xi(\omega)$ is given by

$$\xi(\omega) = \frac{\sqrt{2\omega}\,\int d^3x\,\mathrm{tr}\left[\psi^{\ell'\,\dagger}_{D\,0}((0,\vec{x})\,;P_D)\,\psi^{\ell}_{\bar{B}\,0}(\vec{x})\right]}{\sqrt{M_B}(1+\omega)\sqrt{\int d^3x\,\mathrm{tr}\left|\psi^{\ell'\,\dagger}_{D\,0}((0,\vec{x})\,;P_D)\right|^2}}, \tag{43}$$

with relations given by

$$\begin{aligned}&P_{\bar{B}\,0} = M_B, \qquad P_{D\,0} = M_D v_{D\,0} = M_D\omega, \qquad v_{\bar{B}\,0} = 1, \\ &v_{D\,0} = v_{\bar{B}}\cdot v_D = \gamma = \frac{1}{\sqrt{1-\beta^2}} = \omega, \qquad \beta = \frac{\sqrt{\omega^2-1}}{\omega}.\end{aligned}\tag{44}$$

2) Breit frame

In this frame $\bar{B}$ is moving into the $+z$ direction, D into the $-z$ with the same velocity $\beta/2$ and $\xi(\omega)$ is given by

$$\xi(\omega) = \frac{\int d^3x\,\mathrm{tr}\left[\psi^{\ell'\,\dagger}_{D\,0}((0,\vec{x})\,;P_D)\,\psi^{\ell}_{\bar{B}\,0}((0,\vec{x})\,;P_{\bar{B}})\right]}{\sqrt{\int d^3x\,\mathrm{tr}\left|\psi^{\ell'}_{D\,0}((0,\vec{x})\,;P_D)\right|^2\int d^3x\,\mathrm{tr}\left|\psi^{\ell}_{\bar{B}\,0}((0,\vec{x})\,;P_{\bar{B}})\right|^2}}, \tag{45}$$

with relations given by

$$\begin{aligned}&P_{\bar{B}\,0} = M_{\bar{B}}\gamma, \qquad P_{D\,0} = M_D\gamma, \qquad v_{\bar{B}\,0} = v_{D\,0} = \gamma = \frac{1}{\sqrt{1-(\beta/2)^2}}, \\ &v_{\bar{B}}\cdot v_D = \gamma^2\left(1+\frac{\beta^2}{4}\right) = \omega, \qquad \gamma = \sqrt{\frac{\omega+1}{2}}, \qquad \frac{\beta}{2} = \sqrt{\frac{\omega-1}{\omega+1}},\end{aligned}\tag{46}$$

where one should note that here and below we have used the same notations for velocity and Lorentz factor as in 1) $\bar{B}$ rest frame although they are different quantities as one can see from Eqs. (44, 46).

Now we will calculate this Isgur-Wise function in four cases below. Details are given in Appendix E 1.

1-i) $\bar{B}$ rest frame and $t = x^0 = y^0$

$$\xi(\omega) = \frac{1}{\omega} - \frac{1}{6}\beta^2\omega\tilde{E}_D^2\left\langle r^2\right\rangle + \frac{\beta^2}{4} + \;O(\beta^4), \tag{47}$$

where

$$\left\langle r^2\right\rangle \equiv \int d^3x\,\mathrm{tr}\left[{\psi^{\ell}_{X\,0}}^{\dagger}(\vec{x})r^2\psi^{\ell}_{X\,0}(\vec{x})\right], \qquad \text{for } X = D, \text{ or } \bar{B}, \tag{48}$$

either of which gives the same result since this is the lowest order calculation.

1-ii) $\bar{B}$ rest frame and $t' = x'^0 = y'^0$

$$\xi(\omega) = 1 - \frac{1}{6}\beta^2\omega\tilde{E}_D^2 \left\langle r^2 \right\rangle - \frac{\beta^2}{4} + O(\beta^4), \tag{49}$$

2-i) Breit frame and $t = x^0 = y^0$

$$\xi(\omega) = \gamma^{-2} - \frac{1}{6}(\frac{\beta}{2})^2\gamma^2(\tilde{E}_{\bar{B}} + \tilde{E}_D)^2 \left\langle r^2 \right\rangle + O(\beta^4). \tag{50}$$

2-ii) Breit frame and $t' = x'^0 = y'^0$

$$\xi(\omega) = \gamma^{-2} - \frac{1}{6}(\frac{\beta}{2})^2\gamma^{-2}(\tilde{E}_{\bar{B}} + \tilde{E}_D)^2 \left\langle r^2 \right\rangle + O(\beta^4). \tag{51}$$

Here in the above defined are

$$\tilde{E}_D = M_D - m_c, \qquad \tilde{E}_{\bar{B}} = M_B - m_b. \tag{52}$$

The rhs of Eq. (52) can be expanded in $1/m_Q$ as

$$\tilde{E}_X = M_X - m_Q = \left(m_Q + \sum_{i=0} \frac{C_X^i}{(m_Q)^i} \right) - m_Q = \sum_{i=0} \frac{C_X^i}{(m_Q)^i}, \tag{53}$$

where i-th order in $1/m_Q$ of M_X is given by $C_X^i/(m_Q)^i$, and it is shown in [15] that C_X^i depends only on a light quark mass m_q and hence $C_D^i = C_B^i = C^i(m_q)$ in our case since light quarks are either u or d and we have set $m_u = m_d$. Eq. (53) is for 0^- state when it is written for 1^- coefficients are given by ${C_X^i}'$. All these expressions obtained above have the same form up to the first order in ω in the vicinity of $\omega = 1$, and up to the zeroth order in $1/m_Q$ as given by

$$\xi(\omega) = 1 - \left(\frac{1}{2} + \frac{1}{3}\bar{\Lambda}^2 \left\langle r^2 \right\rangle \right) (\omega - 1), \tag{54}$$

$$\bar{\Lambda} = \lim_{m_Q \to \infty} \tilde{E}_D = \lim_{m_Q \to \infty} \tilde{E}_{\bar{B}} = C^0, \tag{55}$$

that is,

$$\xi(1) = 1, \qquad \xi'(1) = -\frac{1}{2} - \frac{1}{3}\bar{\Lambda}^2 \left\langle r^2 \right\rangle. \tag{56}$$

This form for $\xi(\omega)$ given by Eq. (54) is derived from our semi-relativistic formulation and coincides with those derived from other considerations [24, 27, 29] but we believe that our derivation is the most general derivation and does not depend on any specific model. This form gives the lower bound for $\xi'(1)$ as $-1/2$ or the upper bound for $-\xi'(1)$ as $1/2$ as shown in these papers. All the other form factors, $\xi_-(\omega)$, $\xi_V(\omega)$, and $\xi_{A_i}(\omega)$ for $i = 1 \sim 3$ are similarly calculated and given by Eqs. (41) up to this order $(1/m_Q)^0$.

Using the values of parameters obtained in the former paper [15] to calculate mass spectra of heavy mesons, we can calculate $\xi'(1)$ at the first and second order in $1/m_Q$ as

$$\xi'(1) = -1.44, \tag{57}$$

where we have used the values, $\bar{\Lambda} = 0.752$ GeV and $\left\langle r^2 \right\rangle = 5.009$ GeV^{-2} in [16] (hep-ph/0605019). The several values of $\xi'(1)$ in references are listed in [27].

B. First Order

There is only one kind of the first order corrections, i.e., one from the wave functions. The first order corrections from the wave functions to the form factors defined by Eq. (39) are straightforwardly calculated although a cumbersome work to do. Hence here we do not repeat the similar calculations to the former subsection III A and give only the final results below.

We have calculated all the four cases, 1-i) $\sim$ 2-ii), which show that there are no contributions from the first order corrections of the wave functions to the form factors. Although we try to obtain relativistically invariant results from the rest frame knowledge by applying four different Lorentz-boost frames, the first order corrections in all cases are not invariant but one. That is, the case 2-ii): Breit frame with $t' = x'^0 = y'^0$ gives the relativistic results which coincide with those of [21]. Hence the following results could be a conjecture from our model but we believe that our results are very plausible since they agree with relativistic results which come from a semi-relativistic potential model. Brief derivations of these form factors are given in Appendix E 2.

As stressed in the Introduction, our approach does not use fields and hence there is no other kind of the first orders, i.e., the currents cannot be expanded in $1/m_Q$ in terms of the effective velocity-dependent fields like in the HQET as given in many papers as

$$j_\mu = c^\dagger \gamma^0 \gamma_\mu b + c^\dagger \left(\frac{-i}{2m_c} \gamma^0 \overleftarrow{\not{D}} \gamma_\mu + \frac{i}{2m_b} \gamma^0 \gamma_\mu \overrightarrow{\not{D}} \right) b.$$

Six form factors including zeroth order terms for completeness with a decomposition of Neubert and Rieckert [21] are given by,

$$\xi_+(\omega) = \left[1 + \left(\frac{1}{m_c} + \frac{1}{m_b} \right) \rho_1(\omega) \right] \xi(\omega), \tag{58a}$$

$$\xi_-(\omega) = \left[-\frac{\bar\Lambda}{2} + \rho_4(\omega) \right] \left(\frac{1}{m_c} - \frac{1}{m_b} \right) \xi(\omega), \tag{58b}$$

$$\xi_V(\omega) = \left[1 + \frac{\bar\Lambda}{2} \left(\frac{1}{m_c} + \frac{1}{m_b} \right) + \frac{1}{m_c} \rho_2(\omega) + \frac{1}{m_b} \left(\rho_1(\omega) - \rho_4(\omega) \right) \right] \xi(\omega), \tag{58c}$$

$$\xi_{A_1}(\omega) = \left[1 + \frac{\bar\Lambda}{2} \frac{\omega-1}{\omega+1} \left(\frac{1}{m_c} + \frac{1}{m_b} \right) + \frac{1}{m_c} \rho_2(\omega) + \frac{1}{m_b} \left(\rho_1(\omega) - \frac{\omega-1}{\omega+1} \rho_4(\omega) \right) \right] \xi(\omega), \tag{58d}$$

$$\xi_{A_2}(\omega) = \frac{1}{\omega+1} \frac{1}{m_c} \left[-\bar\Lambda + (\omega+1)\rho_3(\omega) - \rho_4(\omega) \right] \xi(\omega), \tag{58e}$$

$$\begin{aligned} \xi_{A_3}(\omega) = & \left[1 + \frac{\bar\Lambda}{2} \left(\frac{\omega-1}{\omega+1} \frac{1}{m_c} + \frac{1}{m_b} \right) + \frac{1}{m_c} \left(\rho_2(\omega) - \rho_3(\omega) - \frac{1}{\omega+1} \rho_4(\omega) \right) \right. \\ & \left. + \frac{1}{m_b} \left(\rho_1(\omega) - \rho_4(\omega) \right) \right] \xi(\omega). \end{aligned} \tag{58f}$$

where $\xi(\omega)$ is given by Eq. (54) and

$$\rho_1(\omega) = \rho_2(\omega) = -\frac{1}{3} C^1 \bar\Lambda \left\langle r^2 \right\rangle (\omega - 1), \tag{59a}$$

$$\rho_3(\omega) = \rho_4(\omega) = 0. \tag{59b}$$

i.e., there are $1/m_Q$ corrections for ρ_1 and ρ_2 coming from phase factors of the wave functions together with kinetic terms. See Appendix E 2.

IV. CKM MATRIX ELEMENT

A. $\bar{B} \to D\ell\nu$

With the results given in Subsects. III A and III B, we now calculate the CKM matrix element, $|V_{cb}|$. We first evaluate the differential decay rate of the process $\bar{B} \to D\ell\nu$ to extract the value $|V_{cb}|$. The expression of this differential decay rate is given by

$$\frac{d\Gamma}{d\omega} = \frac{G_F^2}{48\pi^3} |V_{cb}|^2 M_D^3 (M_B + M_D)^2 (\omega^2 - 1)^{3/2} \mathcal{F}_D(\omega)^2, \tag{60}$$

where defined are

$$\mathcal{F}_D(\omega)^2 = \xi_+(\omega) - \frac{1-r}{1+r} \xi_-(\omega), \quad r = \frac{M_D}{M_B}. \tag{61}$$

Using the above expressions and the form factors obtained in the former subsections, one can evaluate values of the form factor $\mathcal{F}_D(\omega)$ at the zero recoil and its first derivative.

i) Zeroth order in $1/m_Q$

In this case since $\xi_+(\omega) = \xi_V(\omega) = \xi_{A_1}(\omega) = \xi_{A_3}(\omega) = \xi(\omega)$ and $\xi_-(\omega) = \xi_{A_2}(\omega) = 0$ given by Eqs. (41), we have

$$\mathcal{F}_D(1) = \xi(1) = 1, \qquad {\mathcal{F}_D}'(1) = \xi'(1) = -1.44. \tag{62}$$

ii) First order in $1/m_Q$

In this case Eqs. (58) together with Eqs. (59) give

$$\mathcal{F}_D(1) = \xi_+(1) - \frac{1-r}{1+r}\xi_-(1) = 1.07, \qquad {\mathcal{F}_D}'(1) = -0.875, \tag{63}$$

The combined recent experimental data of CLEO [37] and Belle [38] leads to the value of the product of the form factor and CKM matrix element as,

$$\mathcal{F}_D(1)\,|V_{cb}| = 0.0414 \pm 0.0064.$$

Using our prediction Eq. (63), we find the value of CKM matrix element as

$$|V_{cb}| = 0.0387 \pm 0.0060. \tag{64}$$

B. $\bar{B} \to D^*\ell\nu$

As noted by Neubert [39], the differential decay rate of the process $\bar{B} \to D^*\ell\nu$ is the best quantity to extract the value $|V_{cb}|$. The expression of this differential decay rate is given by

$$\frac{d\Gamma}{d\omega} = \frac{G_F^2}{48\pi^3}\,|V_{cb}|^2\,M_{D^*}^3\,(M_B - M_{D^*})^2\,\sqrt{\omega^2-1}(\omega+1)^2\left[1 + \frac{4\omega}{\omega+1}\frac{1-2\omega r + r^2}{(1-r)^2}\right]\mathcal{F}_{D^*}(\omega)^2, \tag{65}$$

where defined are

$$\begin{aligned}
&(1-r^*)^2\left[1 + \frac{4\omega}{\omega+1}\frac{1-2\omega r^* + r^{*2}}{(1-r^*)^2}\right]\mathcal{F}_{D^*}(\omega)^2 \\
&= 2(1-2\omega r^* + r^{*2})\left(\xi_{A_1}(\omega)^2 + \frac{\omega-1}{\omega+1}\xi_V(\omega)^2\right) + \Big\{(\omega - r)\xi_{A_1}(\omega) \\
&\qquad -(\omega-1)\left[\xi_{A_3}(\omega) + r\xi_{A_2}(\omega)\right]\Big\}^2 \\
&= \left\{2(1-2\omega r^* + r^{*2})\left(1 + \frac{\omega-1}{\omega+1}R_1(\omega)^2\right) + \left[(\omega - r^*) - (\omega-1)R_2(\omega)\right]^2\right\}\xi_{A_1}(\omega)^2,
\end{aligned} \tag{66}$$

and

$$r^* = \frac{M_{D^*}}{M_B}, \qquad R_1(\omega) = \frac{\xi_V(\omega)}{\xi_{A_1}(\omega)}, \qquad R_2(\omega) = \frac{\xi_{A_3}(\omega) + r\xi_{A_2}(\omega)}{\xi_{A_1}(\omega)}. \tag{67}$$

Similarly to IV A, one can evaluate values of the form factor $\mathcal{F}_{D^*}(\omega)$ at the zero recoil and its first derivative.

i) Zeroth order in $1/m_Q$

In this case, we have

$$\mathcal{F}_{D^*}(1) = \xi(1) = 1, \qquad {\mathcal{F}_{D^*}}'(1) = \xi'(1) = -1.44. \tag{68}$$

ii) First order in $1/m_Q$

In this case, we have

$$\mathcal{F}_{D^*}(1) = \xi(1) = 1, \qquad {\mathcal{F}_{D^*}}'(1) = -1.09, \tag{69a}$$

$$R_1(1) = 1.45, \quad R_1(1) = -0.222, \quad R_2(1) = 0.942, \quad R_2'(1) = 0.0286, \tag{69b}$$

where the first equation is consistent with the so-called Luke's theorem [21] and we have used $m_c = 1.032$ GeV, $m_b = 4.639$ GeV, and $r^* = M_{D^*}/M_B = 0.3804$ used in [17] and $C^1 = 0.19022$ in Eq. (59a). These estimated values given by Eqs. (63) and (69) in our paper are consistent with other theoretical values except for $R_2'(1)$ whose list is given in [32]. The value of $R_2'(1)$ is one order of magnitude smaller than the other theoretical values and this has no experimental data yet.

The combined fit of the experimental data from CLEO [40], Belle [41], and BaBar [42] gives

$$\mathcal{F}_{D^*}(1)\,|V_{cb}| = 0.0380 \pm 0.0021.$$

Using our value of $\mathcal{F}_{D^*}(1) = 1$, we have

$$|V_{cb}| = 0.0380 \pm 0.0021. \tag{70}$$

V. SUMMARY AND DISCUSSIONS

In this paper, we have formulated a method how to construct Lorentz-invariant amplitudes by examining four kinds of Lorentz-boosted systems after normalizing those amplitudes. Although all these four are not always consistent with each other for higher orders in $1/m_Q$, they may be corrected by comparing with other methods, for instance, relativistic and kinematic results. Despite of this defect, we believe that this formulation certainly gives a way to calculate reliable amplitudes from wave functions in the rest frame.

As an example of this formulation, we have calculated the semi-leptonic weak form factors up to the first order in $1/m_Q$ following the study done before [34] and using the results obtained in that paper. What we have found in this paper is

1. The Isgur-Wise function has the following form up to the first order in $1/m_Q$ and in $(\omega - 1)$.

$$\xi(\omega) = 1 - \left(\frac{1}{2} + \frac{1}{3}\bar{\Lambda}^2\left\langle r^2\right\rangle\right)(\omega - 1)\,, \qquad \bar{\Lambda} = \bar{\Lambda}_u = \lim_{m_Q\to\infty}\tilde{E}_D = \lim_{m_Q\to\infty}\tilde{E}_B,$$

   and hence

$$\xi(1) = 1, \qquad \xi'(1) = -\frac{1}{2} - \frac{1}{3}\bar{\Lambda}^2\left\langle r^2\right\rangle.$$

   Here since $\bar{\Lambda}$ depends only on light quark mass and we treat only heavy mesons D, D^*, and B which include only u and d light quarks with $m_u = m_d$, the subscript of $\bar{\Lambda}_u$ expresses this fact.

2. We find that there is no contribution from correction terms in the first order of m_Q for the rest frame wave functions to the six form factors. That is, the terms except for the first in Eqs. (27) which include negative as well as positive energy states of a heavy quark do not contribute to the physical quantities.

3. The first order corrections are derived from phase factors of the wave functions and also given by kinetic terms and there are no contributions from the first order corrections to the wave functions. They are explicitly given by Eqs. (58) and (59). That is, in the terminology of Neubert and Rieckert in [21],

$$\rho_1(\omega) \;=\; \rho_2(\omega) = -\frac{1}{3}C^1\bar{\Lambda}\left\langle r^2\right\rangle(\omega - 1)\,, \quad \rho_3(\omega) = \rho_4(\omega) = 0.$$

4. We have calculated the values for the form factor $\mathcal{F}(\omega)$ at the zero recoil and/or their first derivatives up to the first order in $1/m_Q$ as

$$\begin{aligned}
&\mathcal{F}_D(1) = 1.07, \qquad \mathcal{F}_D{}'(1) = -0.875,\\
&\mathcal{F}_{D^*}(1) = \xi(1) = 1, \qquad \mathcal{F}_{D^*}{}'(1) = -1.09,\\
&R_1(1) = 1.45, \quad R_1(1) = -0.222, \quad R_2(1) = 0.942, \quad R_2'(1) = 0.0286,
\end{aligned}$$

   the first equations are obtained by analyzing $\bar{B} \to D\ell\nu$ process and the second by $\bar{B} \to D^*\ell\nu$. These values are consistent with experimental data as well as other theoretical estimates listed in Tables I and II of [32].

5. The above values can be used to estimate the CKM matrix element $|V_{cb}|$ which we have obtained as

$$|V_{cb}| = 0.0387 \pm 0.0060,$$

for the $\bar{B} \to D\ell\nu$ process and

$$|V_{cb}| = 0.0380 \pm 0.0021,$$

for the $\bar{B} \to D^{*}\ell\nu$ process. These values are consistent with the value in PDG [43]

$$|V_{cb}| = 0.0409 \pm 0.0018 \quad \text{(exclusive)}.$$

Here to give theoretical predictions for $|V_{cb}|$ we have neglected theoretical undertainties although experimental errors are taken into account.

We have developed a method to obtain the relativistically invariant results using the rest frame wave functions, and to do this the four different Lorentz-boosted frames are adopted to check the validity of our results. The same form is obtained for the Isgur-Wise function in all four cases up to the zeroth order in $1/m_Q$ and the first order in $(\omega - 1)$, however their first order corrections in $1/m_Q$ are not the same for all four cases. Only the case, written as 2-ii) in the main text, i.e., in the Breit frame with $t' = x'^0 = y'^0$ both for $\bar{B}$ and $D^{(*)}$, gives consistent results with the relativistic ones given by [21].

APPENDIX A: SCHRÖDINGER EQUATION

The Schrödinger equation given by Eq. (3) is derived by calculating

$$\langle X'; P_{X'}\,\{\ell'\}\,|\mathcal{H}|\,X; P_X\,\{\ell\}\rangle = \left\langle X; P_X\,\{\ell\}\left|\mathcal{P}^0\right| X; P_X\,\{\ell\}\right\rangle, \tag{A1}$$

with $\mathcal{P}^0$ being the 0-th component of the four-momentum $\mathcal{P}^\mu$, and by varying in terms of $\psi_{X'}^{\ell'\,*}$. Here the Hamiltonian density is given by

$$\begin{aligned} \mathcal{H} &= \int d^3x\,\{q^{c\dagger}(\vec{x})(\vec{\alpha}_q\cdot\vec{p}_q + \beta_q m_q)q^c(\vec{x}) + Q^\dagger(\vec{x})(\vec{\alpha}_Q\cdot\vec{p}_Q + \beta_Q m_Q)Q(\vec{x})\} \\ &\quad + \int d^3x d^3y\; q^{c\dagger}(\vec{x})\beta_q O_{qi} q^c(\vec{x}) V_{ij}(\vec{x}-\vec{y}) Q^\dagger(\vec{y})\beta_Q O_{Qj} Q(\vec{y}). \end{aligned} \tag{A2}$$

The lhs of Eq. (A1) can be approximated as

$$\begin{aligned} &\langle X'; P_{X'}\,\{\ell'\}\,|\mathcal{H}|\,X; P_X\,\{\ell\}\rangle \\ &= \left\langle X; P_X\,\{\ell\}\left|\int d^3x \int d^3y q_\alpha^{c\,\dagger}(t,\vec{x}) q_\alpha^c(t,\vec{x}) Q_\beta^\dagger(t,\vec{y}) Q_\beta(t,\vec{y})\mathcal{H}\right| X; P_X\,\{\ell\}\right\rangle \\ &\simeq \int d^3x \int d^3y \left\langle X'; P_{X'}\,\{\ell'\}\left|Q_\beta^\dagger(t,\vec{y}) q_\alpha^{c\,\dagger}(t,\vec{x})\right|0\right\rangle \left\langle 0\left|q_\alpha^c(t,\vec{x}) Q_\beta(t,\vec{y})\mathcal{H} Q_\delta^\dagger(t,\vec{y}) q_\gamma^{c\dagger}(t,\vec{x}) q_\gamma^c(t,\vec{x}) Q_\delta(t,\vec{y})\right| X; P_X\,\{\ell\}\right\rangle \\ &\simeq \int d^3x \int d^3y \psi_{X'\,\alpha\,\beta}^{\ell'\,*}\left((0,\vec{x}-\vec{y})\,;P_{X'}\right)\, H_{\alpha\,\gamma,\,\beta\,\delta}\psi_{X\,\gamma\,\delta}^{\ell}\left((0,\vec{x}-\vec{y})\,;P_X\right)\, e^{-i(P_X - P_{X'})\cdot y}, \end{aligned} \tag{A3}$$

where the number operators for q^c and Q have been inserted and the vacuum dominance has been used among the intermediate states. The rhs of Eq. (A1) can be given by

$$\left\langle X'; P_{X'}\,\{\ell'\}\left|\mathcal{P}^0\right| X; P_X\,\{\ell\}\right\rangle = \sqrt{M_X^2 + \vec{P}_X^2}\,\langle X'; P_{X'}\,\{\ell'\}\,|\,X; P_X\,\{\ell\}\rangle\,, \tag{A4}$$

and thus we obtain the Schrödinger equation, Eq. (3).

APPENDIX B: LORENTZ BOOST AND NORMALIZATION

Derivation of Eqs. (13), (15), (22) is also similar to the former subsection A. Setting $\mathcal{H} = 1$ in Eq. (A3), then we obtain

$$\begin{aligned}
&\langle X; P_{X'} \{\ell'\} \mid X; P_X \{\ell\}\rangle \\
&\simeq \int d^3x \int d^3y \, \psi^{\ell' *}_{X\,\alpha\,\beta} \left((0, \vec{x} - \vec{y})\,; P_{X'}\right) \, \psi^{\ell}_{X\,\alpha\,\beta} \left((0, \vec{x} - \vec{y})\,; P_X\right) \, e^{-i(P_X - P_{X'})\cdot y} \\
&= (2\pi)^3 \;\, \delta^3 \left(\vec{P}_{X'} - \vec{P}_X\right) \int d^3z \; \mathrm{tr} \left[\psi^{\ell' \,\dagger}_{X} \left((0, \vec{z})\,; P_X\right) \psi^{\ell}_X \left((0, \vec{z})\,; P_X\right)\right].
\end{aligned} \tag{B1}$$

The lhs is given by

$$\langle X; P'_X \{\ell'\} \mid X; P_X \{\ell\}\rangle = (2\pi)^3 \, 2P_{X\,0} \, \delta^3 \left(\vec{P}_X - \vec{P}'_X\right) \delta_{\ell'\,\ell}, \tag{B2}$$

hence

$$\int d^3z \; \mathrm{tr} \left[\psi^{\ell' \,\dagger}_{X} \left((0, \vec{z})\,; P_X\right) \psi^{\ell}_X \left((0, \vec{z})\,; P_X\right)\right] \simeq 2P_{X\,0} \, \delta_{\ell'\,\ell} \tag{B3}$$

holds.

We have to calculate the normalization of the wave function in the moving frame in two cases, i) $t = x^0 = y^0$ and ii) $t' = x'^0 = y'^0$. To do so, we need to have a relation between the rest frame (RF) and Lorentz boosted (LB) wave functions.

i) $t = x^0 = y^0$ and $x'^0 \neq y'^0$ (equal time in the rest frame)

The relation between RF and LB is derived as follows. By definition

$$\begin{aligned}
&\left\langle 0 \,|\, q^c_\alpha(t, \vec{x}) Q_\beta(t, \vec{y}) |\, X; \left(M_X, \vec{0}\right) \{\ell\} \right\rangle \\
&= G^{-1}_{\alpha\,\gamma} G^{-1}_{\beta\,\delta} \, \left\langle 0 \left| q^c_\gamma(x'^0, \vec{x}') Q_\delta(y'^0, \vec{y}')\right| X; P_X \{\ell\} \right\rangle.
\end{aligned} \tag{B4}$$

Hence

$$\begin{aligned}
&\left\langle 0 \left| \, q^c_\alpha(x'^0, \vec{x}') Q_\beta(y'^0, \vec{y}')\right| X; P_X \{\ell\} \right\rangle \\
&\simeq \psi^\ell_{X\,\alpha\beta} \left((0, \vec{x}' - \vec{y}'); P_X\right) \, \exp\left(i[-M_X t + (M_X - m_Q)\gamma^2\beta(x^3 - y^3)]\right) \\
&= G_{\alpha\,\gamma} G_{\beta\,\delta} \, \left\langle 0 \left| \, q^c_\gamma(t, \vec{x}) Q_\delta(t, \vec{y})\right| X; \left(M_X, \vec{0}\right) \{\ell\} \right\rangle = G_{\alpha\,\gamma} G_{\beta\,\delta} \, \psi^\ell_{X\,\gamma\delta} \left(\vec{x} - \vec{y}\right) \, e^{-iM_X t},
\end{aligned} \tag{B5}$$

where use has been made of the approximation,

$$Q_\beta \left(y'^0, \vec{y}'\right) \simeq \exp\left[-im_Q\gamma \left(y'^0 - x'^0\right)\right] Q_\beta \left(x'^0, \vec{y}'\right). \tag{B6}$$

Rewriting Eq. (B5), one obtains the relation as

$$\psi^\ell_{X\,\alpha\,\beta} \left((0, \vec{x})\,; P_X\right) \simeq G_{\alpha\,\gamma} G_{\beta\,\delta} \, \psi^\ell_{X\,\gamma\,\delta} \left(\left(0, \vec{x}_\perp, \gamma^{-1} x^3\right); \left(M_X, \vec{0}\right)\right) \, e^{i(M_X - m_Q)\gamma\,\beta\, x^3}. \tag{B7}$$

Using this equation, one obtains,

$$\begin{aligned}
&\int \; d^3x \; \psi^{\ell *}_{X\;\alpha\beta} \left((0, \vec{x})\,; P_X\right) \psi^\ell_{X\;\alpha\beta} \left((0, \vec{x})\,; P_X\right) = \gamma \int \; d^3x \; \psi^{X\;\ell\,*}_{\alpha\beta} \left(\vec{x}\right) \left(G^2\right)_{\alpha\gamma} \left(G^2\right)_{\beta\delta} \psi^\ell_{X\;\gamma\delta} \left(\vec{x}\right) \\
&= \gamma \int d^3x \; \mathrm{tr} \left[\psi^{\ell\,\dagger}_X \left(\vec{x}\right) G^2 \psi^\ell_X \left(\vec{x}\right) G^{2\,\mathrm{T}}\right] = 2M\gamma^3.
\end{aligned} \tag{B8}$$

ii) $t' = x'^0 = y'^0$ and $x^0 \neq y^0$ (equal time in the moving frame)

Likewise in the case i) the relation between RF and LB is, in this case, given by

$$\psi^\ell_{X\,\alpha\,\beta} \left((0, \vec{x})\,; P_X\right) \simeq G_{\alpha\,\gamma} G_{\beta\,\delta} \, \psi^\ell_{X\,\gamma\,\delta} \left(\left(0, \vec{x}_\perp, \gamma x^3\right); \left(M_X, \vec{0}\right)\right) \, e^{i(M_X - m_Q)\gamma\,\beta\, x^3}. \tag{B9}$$

Using Eq. (B9) one obtains,

$$\int d^3x\, \psi^{\ell\,*}_{X\,\alpha\beta}\left((0,\vec{x})\,;P_X\right)\psi^{\ell}_{X\,\alpha\beta}\left((0,\vec{x})\,;P_X\right) = \gamma^{-1}\int d^3x\, \psi^{\ell\,*}_{X\,\alpha\beta}\left(\vec{x}\right)\left(G^2\right)_{\alpha\gamma}\left(G^2\right)_{\beta\delta}\psi^{\ell}_{X\,\gamma\delta}\left(\vec{x}\right)$$
$$= \gamma^{-1}\int d^3x\ \mathrm{tr}\left[\psi^{\ell\,\dagger}_{X}\left(\vec{x}\right)G^2\psi^{\ell}_{X}\left(\vec{x}\right)G^{2\,\mathrm{T}}\right] = 2M\gamma. \tag{B10}$$

Here use has been made of,

$$\frac{1}{2}\mathrm{tr}\left[\begin{pmatrix} 0 & u_{-1}(r) \\ 0 & -i(\vec{\sigma}\cdot\vec{n})v_{-1}(r)\end{pmatrix}^{\dagger} G^2 \begin{pmatrix} 0 & u_{-1}(r) \\ 0 & -i(\vec{\sigma}\cdot\vec{n})v_{-1}(r)\end{pmatrix} G^{2\,\mathrm{T}}\right]$$
$$= \frac{\gamma^2}{2}\,\mathrm{tr}\left[\begin{pmatrix} 0 & 0 \\ u_{-1}(r) & i(\vec{\sigma}\cdot\vec{n})v_{-1}(r)\end{pmatrix}\begin{pmatrix} 1 & \sigma^3\beta \\ \sigma^3\beta & 1\end{pmatrix}\begin{pmatrix} 0 & u_{-1}(r) \\ 0 & -i(\vec{\sigma}\cdot\vec{n})v_{-1}(r)\end{pmatrix}\begin{pmatrix} 1 & \sigma^3\beta \\ \sigma^3\beta & 1\end{pmatrix}\right]$$
$$= \gamma^2\left(u^2_{-1}+v^2_{-1}\right). \tag{B11}$$

APPENDIX C: WAVE FUNCTIONS

The wave function is generally defined as

$$\begin{aligned}\langle 0\,|q^c_\alpha(t,\vec{x})\,Q_\beta(t,\vec{y})|\,X;P_X\,\{\ell\}\rangle &= \langle 0\,|q^c_\alpha(0,\eta(\vec{x}-\vec{y}))\,Q_\beta(0,\zeta(\vec{y}-\vec{x}))|\,X;P_X\,\{\ell\}\rangle\; e^{-iP^0_X t} \\ &\equiv \psi^{\ell}_{X\,\alpha\,\beta}\left((0,\vec{x}-\vec{y})\,;P_X\right)\, e^{-iP_X\cdot r},\end{aligned} \tag{C1}$$

with

$$r^\mu = \zeta x^\mu + \eta y^\mu, \quad \zeta+\eta = 1. \tag{C2}$$

Here we adopt without loss of generality

$$\zeta = 0, \qquad \eta = 1.$$

The Foldy-Wouthuysen-Tani transformation and charge conjugation is defined as

$$U_{\mathrm{FWT}}\left(p\right) = \exp\left(W\left(p\right)\,\vec{\gamma}_Q\cdot\hat{\vec{p}}\right) = \cos W + \vec{\gamma}_Q\cdot\hat{\vec{p}}\,\sin W, \tag{C3a}$$
$$\hat{\vec{p}} = \frac{\vec{p}}{p}, \quad \tan W\left(p\right) = \frac{p}{m_Q+E}, \quad E = \sqrt{\vec{p}^{\;2}+m_Q^2}, \tag{C3b}$$
$$U_c = i\,\gamma^0_Q\gamma^2_Q = -U_c^{-1}. \tag{C3c}$$

Here $\vec{p} = \vec{p}_Q$ is an initial momentum of the heavy quark and $U_{\mathrm{FWT}}\left(p\right)$ operates on heavy quarks, henceforth the color, $N_c = 3$, is neglected since the form of the wave function is the same for all colors, ℓ stands for a set of quantum numbers, j, m, and k, and

$$\Psi^k_{j\,m}(\vec{x}) = \frac{1}{r}\begin{pmatrix} u_k(r) \\ -i\,v_k(r)\,(\vec{n}\cdot\vec{\sigma})\end{pmatrix}\; y^k_{j\,m}(\Omega), \tag{C4}$$

where $r = |\vec{x}|$, $\vec{n} = \vec{x}/r$, j is a total angular momentum of a meson, m is its z component, k is a quantum number which takes only values, $k = \pm j,\ \pm(j+1)$ and $\neq 0$, whose operator form is given by $\hat{k} = -\beta_q\left(\vec{\Sigma}_q\cdot\vec{\ell}+1\right)$. The scalar functions, $u_k(r)$ and $v_k(r)$ are polynomials of a radial variable r and satisfy

$$\int dr\left(u^2_k(r)+v^2_k(r)\right) = 1. \tag{C5}$$

$y^k_{j\,m}(\Omega)$ are functions of angles and spinors of a total angular momentum, $\vec{j} = \vec{\ell}+\vec{s}_q+\vec{s}_Q$ with $\vec{\ell} = -i\vec{r}\times\nabla$, whose first few explicit forms are given by

$$y^{-1}_{00} = \frac{1}{\sqrt{4\pi}}, \quad y^{-1}_{1m} = \frac{1}{\sqrt{4\pi}}\sigma^m,$$
$$y^2_{1m} = \frac{-1}{\sqrt{4\pi}}\frac{3}{\sqrt{6}}\left(n^i n^m - \frac{1}{3}\delta^{i\,m}\right)\sigma^i, \tag{C6}$$

and satisfy some relation and the normalization condition as

$$y_{j\,m}^{-k} = -\,(\vec{n}\cdot\vec{\sigma})\,y_{j\,m}^{k}, \tag{C7}$$

$$\frac{1}{2}\mathrm{tr}\left(\int d\Omega\; {y_{j'\,m'}^{k'}}^{\dagger}(\Omega)\; y_{j\,m}^{k}(\Omega)\right) = \delta^{k\,k'}\delta_{j\,j'}\delta_{m\,m'}. \tag{C8}$$

The relative phases among $y_{j\,m}^{k}$ given by Eq. (C6) are determined so that they give the correct relative phases among form factors which are determined by Eq. (39a∼39c). From now on whenever the trace of $y_{j\,m}^{k}$ occurs it is understood to be in a sense of Eq. (C8).

The leading order pseudoscalar state (0^-) corresponds to $\ell = (k=-1,\ j=m=0)$ and hence we have the wave function,

$$\psi_{X\,0}^{\ell}(\vec{x}) = \sqrt{M_X}\,\begin{pmatrix} 0 & \Psi_{0\,0}^{-1}(\vec{x}) \end{pmatrix} = \sqrt{\frac{M_X}{4\pi}}\frac{1}{r}\begin{pmatrix} 0 & u_{-1}(r) \\ 0 & -i(\vec{n}\cdot\vec{\sigma})v_{-1}(r)\end{pmatrix}, \tag{C9}$$

with $X = D$ or B. On the other hand the leading order vector state (1^-) has a set of quantum number, $\ell = (k = -1,\ j = 1)$, and is given by

$$\psi_{X\,0}^{\ell}(\vec{x}) = \sqrt{M_X}\,\begin{pmatrix} 0 & \Psi_{1\,m}^{-1}(\vec{x})\epsilon^m \end{pmatrix} = \sqrt{\frac{M_X}{4\pi}}\frac{1}{r}\begin{pmatrix} 0 & u_{-1}(r) \\ 0 & -i(\vec{n}\cdot\vec{\sigma})v_{-1}(r)\end{pmatrix}(\vec{\epsilon}\cdot\vec{\sigma}), \tag{C10}$$

with ϵ^m being a polarization vector, $\vec{\epsilon}^{\,2} = -1$, and $X = D^*$ or B^*.

APPENDIX D: MATRIX ELEMENTS

The light anti-quark is treated as a spectator. That is, it does not interact with other particles except for gluons represented by potentials in our model. The heavy quark has a weak vertex and its current is in general given by

$$j(\vec{x},t) = Q_{X'\,\alpha}^{\dagger}(t,\vec{x})\,\mathcal{O}_{\alpha\,\beta}\,Q_{X\,\beta}(t,\vec{x}), \tag{D1}$$

where $\mathcal{O}$ is some four by four matrix. By inserting the number operator of the anti-quark and by assuming the vacuum dominance among intermediate states, we obtain the formula for the matrix element of the above current, Eq. (D1), between heavy mesons as follows.

$$\begin{aligned}
&\langle X'; P_{X'}, \{\ell'\}\,|\,j(t,\vec{x})|\,X; P_X\,\{\ell\}\rangle \\
=\;& \mathcal{O}_{\alpha\,\beta}\left\langle X'; P_{X'}\,\{\ell'\}\left|\,Q_{X'\,\alpha}^{\dagger}(t,\vec{x})\int d^3y\,{q_\gamma^c}^{\dagger}(t,\vec{y})\,q_\gamma^c(t,\vec{y})\,Q_{X\,\beta}(t,\vec{x})\right| X; P_X\,\{\ell\}\right\rangle \\
\simeq\;& \int d^3y\,\mathcal{O}_{\alpha\,\beta}\left\langle X'; P_{X'}\,\{\ell'\}\left|\,Q_{X'\,\alpha}^{\dagger}(t,\vec{x}){q_\gamma^c}^{\dagger}(t,\vec{y})\right|0\right\rangle\langle 0\,|\,q_\gamma^c(t,\vec{y})\,Q_{X\,\beta}(t,\vec{x})\,|\,X; P_X\,\{\ell\}\rangle \\
=\;& \int d^3y\;\mathrm{tr}\left[{\psi_{X'}^{\ell'}}^{\dagger}(\vec{y}-\vec{x}; P_{X'})\;\psi_X^{\ell}(\vec{y}-\vec{x}; P_X)\;\mathcal{O}^{\mathrm{T}}\right]\,e^{-i(P_X-P_{X'})\cdot x},
\end{aligned} \tag{D2}$$

or

$$\left\langle X'; P_{X'}\,\{\ell'\}\left|\,j(0,\vec{0})\right| X; P_X\,\{\ell\}\right\rangle = \int d^3x\;\mathrm{tr}\left[{\psi_{X'}^{\ell'}}^{\dagger}(\vec{x}; P_{X'})\;\psi_X^{\ell}(\vec{x}; P_X)\;\mathcal{O}^{\mathrm{T}}\right], \tag{D3}$$

where in Eq. (D2) we have assumed the vacuum dominance among intermediate states to make some approximations.

APPENDIX E: SEMI-LEPTONIC WEAK FORM FACTOR

In this Appendix, we have used the Foldy-Wouthuysen-Tani transformed wave functions, $\psi_{X\;\mathrm{FWT}}^{\ell}$, instead of ψ_X^{ℓ} although the subscript FWT is omitted for simplicity.

1. Zeroth Order (Isgur-Wise Function)

Now we will show the derivations of how to calculate the Isgur-Wise function given in the main text in four cases below.

1-i) $\bar{B}$ rest frame and $t = x^0 = y^0$

$$
\begin{aligned}
\xi(\omega) &= \frac{2\sqrt{\omega}}{(1+\omega)\sqrt{2M_B}} \frac{\int d^3x \,\mathrm{tr}\left[\psi^{\ell'\,\dagger}_{D\,0}\left((0,\vec{x})\,;P_D\right)\psi^{\ell}_{\bar{B}\,0}\left((0,\vec{x})\,;P\right)\right]}{\sqrt{\int d^3x \,\mathrm{tr}\left|\psi^{\ell'}_{D\,0}\left((0,\vec{x})\,;P_D\right)\right|^2}} \\
&= \frac{2\sqrt{\omega}}{(1+\omega)\sqrt{2M_B 2M_D\gamma^3}} \int \ d^3x \ G^*_{\alpha\gamma}G^*_{\beta\delta}\psi^{\ell'\,*}_{D\,0\,\gamma\delta}\left(\left(0,\vec{x}_\perp,\gamma^{-1}x^3\right);\left(M_D,\vec{0}\right)\right) \\
&\qquad \times e^{-i(M_D-m_c)\gamma V x^3}\psi^{\ell}_{\bar{B}\,0\,\alpha\beta}\left(\vec{x}\right) \\
&= \frac{2\sqrt{\omega}}{2\overline{M}(1+\omega)\gamma^{3/2}} \int d^3x \ \left\{\psi^{\ell'\,*}_{D\,0\,\alpha\beta}\left(\vec{x}\right) + (\gamma^{-1}-1)x^3\frac{\partial}{\partial x^3}\psi^{\ell'\,*}_{D\,0\,\alpha\beta}\left(\vec{x}\right)\right\} \\
&\qquad \times G_{\alpha\gamma}G_{\beta\delta}\psi^{\ell}_{\bar{B}\,0\,\gamma\delta}\left(\vec{x}\right)e^{-i(M_D-m_c)\gamma V x^3} + \ O(\beta^4) \\
&= \frac{1}{\omega} - \frac{1}{6}\beta^2\omega\tilde{E}_D^2\left\langle r^2\right\rangle - \frac{\beta^2}{2}\frac{1}{2\overline{M}}\int d^3x \ x^3\left(\frac{\partial}{\partial x^3}\psi^{\ell'\,*}_{D\,0\,\alpha\beta}\left(\vec{x}\right)\right)\psi^{\ell}_{B\,0\,\alpha\beta}\left(\vec{x}\right) + \ O(\beta^4) \\
&= \frac{1}{\omega} - \frac{1}{6}\beta^2\omega\tilde{E}_D^2\left\langle r^2\right\rangle + \frac{\beta^2}{4} + \ O(\beta^4),
\end{aligned} \tag{E1}
$$

where we have used

$$
\begin{aligned}
&\int d^3x \ x^3\left(\frac{\partial}{\partial x^3}\psi^{\ell'\,*}_{D\,0\,\alpha\beta}\left(\vec{x}\right)\right)\psi^{\ell}_{\bar{B}\,0\,\alpha\beta}\left(\vec{x}\right) \\
&= \frac{2\overline{M}}{4\pi}\int d^3x \ \frac{x^3}{2}\mathrm{tr}\ \left[\frac{\partial}{\partial x^3}\left\{\frac{1}{r}\begin{pmatrix} 0 & 0 \\ u_{-1} & i(\vec{n}\cdot\vec{\sigma})\,v_{-1}\end{pmatrix}\right\}\frac{1}{r}\begin{pmatrix} 0 & u_{-1} \\ 0 & -i(\vec{n}\cdot\vec{\sigma})v_{-1}\end{pmatrix}\right] \\
&= \frac{2\overline{M}}{4\pi}\int d^3x \ \frac{x^3}{2}\mathrm{tr}\ \left[\left\{\frac{\partial}{\partial x^3}\left(\frac{u_{-1}}{r}\right)\right\}\frac{u_{-1}}{r} + \left\{\frac{\partial}{\partial x^3}(\vec{n}\cdot\vec{\sigma})\,\frac{v_{-1}}{r}\right\}(\vec{n}\cdot\vec{\sigma})\,\frac{v_{-1}}{r}\right] \\
&= \frac{2\overline{M}}{4\pi}\int d^3x \ \frac{x^3}{4}\frac{\partial}{\partial x^3}\mathrm{tr}\ \left[\left(\frac{u_{-1}}{r}\right)^2 + (\vec{n}\cdot\vec{\sigma})^2\left(\frac{v_{-1}}{r}\right)^2\right] = -\overline{M}\int_0^\infty dr\ \left(u_{-1}^2 + v_{-1}^2\right) \\
&= -\overline{M},
\end{aligned} \tag{E2}
$$

with $\overline{M} = \sqrt{M_B M_D}$.

1-ii) $\bar{B}$ rest frame and $t' = x'^0 = y'^0$

$$
\begin{aligned}
\xi(\omega) &= \frac{2\sqrt{\omega}}{(1+\omega)\sqrt{2M_B 2M_D\gamma}} \int \ d^3x \ G^*_{\alpha\gamma}G^*_{\beta\delta}\psi^{\ell'\,*}_{D\,0\,\gamma\delta}\left(\left(0,\vec{x}_\perp,\gamma x^3\right);\left(M_D,\vec{0}\right)\right) \\
&\qquad \psi^{\ell}_{\bar{B}\,0\,\alpha\beta}\left(\vec{x}\right)\times e^{-i(M_D-m_c)\gamma\beta x^3} \\
&= \frac{2\sqrt{\omega}}{2\overline{M}(1+\omega)\gamma^{1/2}} \int d^3x \ \left\{\psi^{\ell'\,*}_{D\,0\,\alpha\beta}\left(\vec{x}\right) + (\gamma-1)\,x^3\frac{\partial}{\partial x^3}\psi^{\ell'\,*}_{D\,0\,\alpha\beta}\left(\vec{x}\right)\right\} \\
&\qquad \times G_{\alpha\gamma}G_{\beta\delta}\psi^{\ell}_{\bar{B}\,0\,\gamma\delta}\left(\vec{x}\right)e^{-i(M_D-m_c)\gamma\beta x^3} + \ O(\beta^4) \\
&= 1 - \frac{1}{6}\beta^2\omega\tilde{E}_D^2\left\langle r^2\right\rangle + \frac{\beta^2}{2}\frac{1}{2\overline{M}}\int d^3x \ x^3\left(\frac{\partial}{\partial x^3}\psi^{\ell'\,*}_{D\,0\,\alpha\beta}\left(\vec{x}\right)\right)\psi^{\ell}_{\bar{B}\,0\,\alpha\beta}\left(\vec{x}\right) + \ O(\beta^4) \\
&= 1 - \frac{1}{6}\beta^2\omega\tilde{E}_D^2\left\langle r^2\right\rangle - \frac{\beta^2}{4} + \ O(\beta^4),
\end{aligned} \tag{E3}
$$

2-i) Breit frame and $t = x^0 = y^0$

$$
\begin{aligned}
\xi(\omega) &= \frac{\int d^3x \,\mathrm{tr}\left[\psi^{\ell'\,\dagger}_{D\,0}\left((0,\vec{x})\,;P_D\right)\psi^{\ell}_{B\,0}\left((0,\vec{x})\,;P_B\right)\right]}{\sqrt{\int d^3x \,\mathrm{tr}\left|\psi^{\ell'}_{D\,0}\left((0,\vec{x})\,;P_D\right)\right|^2 \int d^3x \,\mathrm{tr}\left|\psi^{\ell}_{B\,0}\left((0,\vec{x})\,;P_B\right)\right|^2}} \\
&= \frac{1}{\sqrt{2M_B\gamma^3 2M_D\gamma^3}}\int d^3x\; G^*_{\alpha\gamma}G^*_{\beta\delta}\psi^{\ell'\,*}_{D\,0\,\gamma\delta}\left(\left(0,\vec{x}_\perp,\gamma^{-1}x^3\right);\left(M_D,\vec{0}\right)\right)e^{-i(M_D-m_c)\gamma\frac{\beta}{2}x^3} \\
&\qquad \times G^{-1}_{\alpha\epsilon}G^{-1}_{\beta\tau}\psi^{\ell}_{\bar{B}\,0\,\epsilon\tau}\left(\left(0,\vec{x}_\perp,\gamma^{-1}x^3\right);\left(M_B,\vec{0}\right)\right)e^{-i(M_B-m_b)\gamma\frac{\beta}{2}x^3} \\
&= \frac{\gamma}{2\overline{M}\gamma^3}\int d^3z\;\psi^{\ell'\,*}_{D\,0\,\alpha\beta}\left(\vec{z}\right)\psi^{\ell}_{\bar{B}\,0\,\alpha\beta}\left(\vec{z}\right)e^{-i(\tilde{E}_B+\tilde{E}_D)\gamma^2\frac{\beta}{2}z^3} \\
&= \gamma^{-2}-\frac{1}{6}(\frac{\beta}{2})^2\gamma^2(\tilde{E}_B+\tilde{E}_D)^2\left\langle r^2\right\rangle+O(\beta^4).
\end{aligned}
\tag{E4}
$$

2-ii) Breit frame and $t'=x'^0=y'^0$

$$
\begin{aligned}
\xi(\omega) &= \frac{1}{\sqrt{2M_B\gamma 2M_D\gamma}}\int d^3x\; G^*_{\alpha\gamma}G^*_{\beta\delta}\psi^{\ell'\,*}_{D\,0\,\gamma\delta}\left(\left(0,\vec{x}_\perp,\gamma^{-1}x^3\right);\left(M_D,\vec{0}\right)\right) \\
&\qquad \times G^{-1}_{\alpha\epsilon}G^{-1}_{\beta\tau}\psi^{\ell}_{\bar{B}\,0\,\epsilon\tau}\left(\left(0,\vec{x}_\perp,\gamma^{-1}x^3\right);\left(M_B,\vec{0}\right)\right)e^{-i(\tilde{E}_B+\tilde{E}_D)\gamma\frac{\beta}{2}x^3} \\
&= \frac{\gamma^{-1}}{2\overline{M}\gamma}\int d^3z\;\psi^{\ell'\,*}_{D\,0\,\alpha\beta}\left(\vec{z}\right)\psi^{\ell}_{\bar{B}\,0\,\alpha\beta}\left(\vec{z}\right)e^{-i(\tilde{E}_B+\tilde{E}_D)\frac{\beta}{2}z^3} \\
&= \gamma^{-2}-\frac{1}{6}(\frac{\beta}{2})^2\gamma^{-2}(\tilde{E}_B+\tilde{E}_D)^2\left\langle r^2\right\rangle+O(\beta^4).
\end{aligned}
\tag{E5}
$$

2. First Order

Here in this Appendix we will show how to calculate the first order corrections to the form factors by using Eqs. (25∼27, 29) and equations in Appendix C. Only the case 2-ii) gives six form factors which conform with those of Neubert and Rieckert [21] among four different Lorentz frames mentioned in the former subsection.

$$
\begin{aligned}
\xi_+(\omega) &= \frac{1}{2\overline{M}\gamma^2}\int d^3z\;\mathrm{tr}\left(\psi^{\ell\,\dagger}_{D}(\vec{z})\psi^{\ell}_{\bar{B}}(\vec{z})U_B^{-1}G^{-1}G^*U_D\right)e^{-2i\Lambda\frac{\beta}{2}z^3} \\
&= \frac{1}{2\overline{M}\gamma^2}\int d^3z\;\mathrm{tr}\left(\psi^{\ell\,\dagger}_{D\,0}(\vec{z})\psi^{\ell}_{\bar{B}\,0}(\vec{z})\left(1+\frac{1}{2}\left[\frac{1}{m_c}\vec{\gamma}\cdot\vec{p}_c-\frac{1}{m_b}\vec{\gamma}\cdot\vec{p}_b\right]\right)\right)e^{-2i\Lambda\frac{\beta}{2}z^3} \\
&= 1-\left(\frac{1}{2}+\frac{1}{3}\Lambda^2\left\langle r^2\right\rangle\right)(\omega-1)=1-\left[\frac{1}{2}+\frac{1}{3}\left(\bar{\Lambda}+C^1\left(\frac{1}{m_c}+\frac{1}{m_b}\right)^2\cdots\right)\left\langle r^2\right\rangle\right](\omega-1) \\
&\sim \left[1-\frac{2}{3}\bar{\Lambda}C^1\left\langle r^2\right\rangle(\omega-1)\left(\frac{1}{m_c}+\frac{1}{m_b}\right)\right]\left[1-\left(\frac{1}{2}+\frac{1}{3}\bar{\Lambda}^2\right)\left\langle r^2\right\rangle(\omega-1)\right] \\
&= \left[1-\frac{2}{3}\bar{\Lambda}C^1\left\langle r^2\right\rangle(\omega-1)\left(\frac{1}{m_c}+\frac{1}{m_b}\right)\right]\xi(\omega),
\end{aligned}
\tag{E6}
$$

where U_X for $X=\bar{B}$ and/or D are defined by Eqs. (16, C3) and $\overline{M}=\sqrt{M_BM_D}$. Here Λ is defined by

$$
\Lambda=\frac{\tilde{E}_D+\tilde{E}_{\bar{B}}}{2}=\frac{(E_D-m_c)+(E_{\bar{B}}-m_b)}{2}=C^0+\frac{C^1}{2}\left(\frac{1}{m_c}+\frac{1}{m_b}\right)+O\left(1/(m_Q)^2\right),
\tag{E7}
$$

which comes from expansion of the phase factor $\exp\left(-2i\Lambda\frac{\beta}{2}z^3\right)$. Here $C^0=\bar{\Lambda}=\bar{\Lambda}_u$ and C^1 is defined in Eq.(53) and depends only on $m_q=m_u=m_d$. Comparing the result with Eq.(58a), we obtain

$$
\xi(\omega) = 1-\left(\frac{1}{2}+\frac{1}{3}\bar{\Lambda}^2\right)\left\langle r^2\right\rangle(\omega-1),
\tag{E8}
$$

$$
\xi_+(\omega) = \left[1+\left(\frac{1}{m_c}+\frac{1}{m_b}\right)\rho_1(\omega)\right]\xi(\omega),\quad \rho_1(\omega)=-\frac{1}{3}\bar{\Lambda}C^1\left\langle r^2\right\rangle(\omega-1).
\tag{E9}
$$

Likewise other five form factors and $\rho_i(\omega)$ are given by the following.

$$\begin{aligned}
\xi_-(\omega) &= \frac{1}{2\overline{M}\gamma^2\frac{\beta}{2}}\int d^3z\ \mathrm{tr}\left({\psi^\ell_D}^\dagger(\vec{z})\psi^\ell_{\bar{B}}(\vec{z})U_B^{-1}G^{-1}{\alpha^3}^{\mathrm{T}}G^*U_D\right)\ e^{-2i\Lambda\frac{\beta}{2}z^3} \\
&= -\frac{1}{2\overline{M}\gamma^2}\int d^3z\ \mathrm{tr}\left({\psi^\ell_{D\,0}}^\dagger(\vec{z})\psi^\ell_{\bar{B}\,0}(\vec{z})\left(\alpha^3+\frac{1}{2}\left[\frac{1}{m_c}p_c^3+\frac{1}{m_b}p_b^3\right]\gamma^0\right)\right)\ e^{-2i\Lambda\frac{\beta}{2}z^3} \\
&= \left(-\frac{\bar{\Lambda}}{2}+\rho_4(\omega)\right)\left(\frac{1}{m_c}-\frac{1}{m_b}\right)\xi(\omega), \qquad \text{(E10)}\\
\rho_4(\omega) &= C^1\left(-\frac{1}{4}+\frac{1}{6}\bar{\Lambda}^2\left\langle r^2\right\rangle(\omega-1)\right)\left(\frac{1}{m_c}+\frac{1}{m_b}\right)=0+O(1/m_Q). \qquad \text{(E11)}
\end{aligned}$$

$$\begin{aligned}
\xi_V(\omega) &= \frac{1}{2\overline{M}^*\gamma^2\frac{\beta}{2}(-i\epsilon^{2*})}\int d^3z\ \mathrm{tr}\left({\psi^\ell_{D^*}}^\dagger(\vec{z})\psi^\ell_{\bar{B}}(\vec{z})U_B^{-1}G^{-1}{\alpha^1}^{\mathrm{T}}G^*U_{D^*}\right)\ e^{-2i\Lambda'\frac{\beta}{2}z^3} \\
&= -\frac{1}{2\overline{M}^*\gamma\frac{\beta}{2}(-i\epsilon^{2*})}\int d^3z\ \mathrm{tr}\left({\psi^\ell_{D^*\,0}}^\dagger(\vec{z})\psi^\ell_{\bar{B}\,0}(\vec{z})\left(\alpha^1+i\frac{\beta}{2}\Sigma^2+\frac{i}{2}\left[\frac{1}{m_c}p_c^3-\frac{1}{m_b}p_b^3\right]\gamma^0\Sigma^2\right)\right) \\
&\quad\times e^{-2i\Lambda'\frac{\beta}{2}z^3} \\
&= \left[1+\frac{\Lambda'}{2}\left(\frac{1}{m_c}+\frac{1}{m_b}\right)\right]\xi(\omega) \\
&= \left[1+\frac{\bar{\Lambda}}{2}\left(\frac{1}{m_c}+\frac{1}{m_b}\right)+\left(\frac{1}{m_c}+\frac{1}{m_b}\right)\rho_1(\omega)\right]\xi(\omega), \qquad \text{(E12)}\\
\rho_2(\omega) &= \rho_1(\omega), \qquad \rho_4(\omega)=0, \qquad \text{(E13)}
\end{aligned}$$

where $\vec{\Sigma}=\begin{pmatrix}\vec{\sigma} & 0\\ 0 & \vec{\sigma}\end{pmatrix}$, $\vec{\epsilon}$ is a static polarization vector for D^*, and $\overline{M}^*=\sqrt{M_BM_{D^*}}$. Here we define

$$\Lambda'=\frac{\tilde{E}_{D^*}+\tilde{E}_{\bar{B}}}{2}=\frac{(E_{D^*}-m_c)+(E_{\bar{B}}-m_b)}{2}=C^0+\frac{{C^1}'}{2m_c}+\frac{C^1}{2m_b}+O\left(1/(m_Q)^2\right), \qquad \text{(E14)}$$

with ${C^0}'=C^0=\bar{\Lambda}=\bar{\Lambda}_u$.

$$\begin{aligned}
\xi_{A_1}(\omega) &= \frac{1}{2\overline{M}^*\gamma^2\epsilon^{1*}}\int d^3z\ \mathrm{tr}\left({\psi^\ell_{D^*}}^\dagger(\vec{z})\psi^\ell_{\bar{B}}(\vec{z})U_B^{-1}G^{-1}\gamma_5^{\mathrm{T}}{\alpha^1}^{\mathrm{T}}G^*U_D\right)\ e^{-2i\Lambda'\frac{\beta}{2}z^3} \\
&= \frac{1}{2\overline{M}^*\gamma^2\epsilon^{1*}}\int d^3z\ \mathrm{tr}\left({\psi^\ell_{D^*\,0}}^\dagger(\vec{z})\psi^\ell_{\bar{B}\,0}(\vec{z})\left(\alpha^1+i\frac{\beta}{2}\Sigma^2+\frac{1}{2}\left[\frac{1}{m_c}p_c^3-\frac{1}{m_b}p_b^3\right]\gamma^0\Sigma^1\right)\right) \\
&\quad\times e^{-2i\Lambda'\frac{\beta}{2}z^3} \\
&= \left[1+\frac{\Lambda'}{2}\frac{\omega-1}{\omega+1}\left(\frac{1}{m_c}+\frac{1}{m_b}\right)\right]\xi(\omega) \\
&= \left[1+\frac{\bar{\Lambda}}{2}\frac{\omega-1}{\omega+1}\left(\frac{1}{m_c}+\frac{1}{m_b}\right)+\left(\frac{1}{m_c}+\frac{1}{m_b}\right)\rho_2(\omega)\right]\xi(\omega), \qquad \text{(E15)}\\
\rho_2(\omega) &= \rho_1(\omega), \qquad \rho_4(\omega)=0. \qquad \text{(E16)}
\end{aligned}$$

$$\begin{aligned}
-\,\xi_{A_1}(\omega) &+ \xi_{A_2}(\omega)+\xi_{A_3}(\omega)=\frac{1}{2\overline{M}^*\gamma^4\frac{\beta}{2}\epsilon^{3*}}\int d^3z\ \mathrm{tr}\left({\psi^\ell_{D^*}}^\dagger(\vec{z})\psi^\ell_{\bar{B}}(\vec{z})U_B^{-1}G^{-1}\gamma_5^{\mathrm{T}}G^*U_{D^*}\right)\ e^{-2i\Lambda'\frac{\beta}{2}z^3} \\
&= -\frac{1}{2\overline{M}^*\gamma^4\frac{\beta}{2}\epsilon^{3*}}\int d^3z\ \mathrm{tr}\left({\psi^\ell_{D^*\,0}}^\dagger(\vec{z})\psi^\ell_{\bar{B}\,0}(\vec{z})\gamma_5\left(1+\frac{1}{2}\left[\frac{1}{m_c}p_c^3+\frac{1}{m_b}p_b^3\right]\right)\right)e^{-2i\Lambda'\frac{\beta}{2}z^3} \\
&= \frac{\Lambda'}{2}\left(\frac{1}{m_c}-\frac{1}{m_b}\right)\xi(\omega). \qquad \text{(E17)}
\end{aligned}$$

$$
\begin{aligned}
\xi_{A_1}(\omega) \; &+ \; \frac{\omega-1}{\omega+1}\left(\xi_{A_2}(\omega)-\xi_{A_3}(\omega)\right) = \frac{1}{2\overline{M}^{*}\gamma^4\epsilon^{3\,*}}\int d^3z \;\mathrm{tr}\left({\psi^{\ell}_{D^*}}^{\dagger}(\vec{z})\psi^{\ell}_{\bar{B}}(\vec{z})U_B^{-1}G^{-1}\gamma_5^{\mathrm{T}}{\alpha^{3}}^{\mathrm{T}}G^{*}U_{D^*}\right) \\
&\times e^{-2i\Lambda'\frac{\beta}{2}z^3} \\
&= -\frac{1}{2\overline{M}^{*}\gamma\epsilon^{3\,*}}\int d^3z \;\mathrm{tr}\left({\psi^{\ell}_{D^*\,0}}^{\dagger}(\vec{z})\psi^{\ell}_{\bar{B}\,0}(\vec{z})\left(\Sigma^3-\frac{1}{2}\left[\frac{1}{m_c}p_c^3-\frac{1}{m_b}p_b^3\right]\gamma_5\gamma^0\right)\right)e^{-2i\Lambda'\frac{\beta}{2}z^3} \\
&= \left(1-\frac{\omega-1}{\omega+1}\right)\xi(\omega). \qquad \text{(E18)}
\end{aligned}
$$

From these we derive

$$
\xi_{A_2}(\omega) \;=\; -\frac{\bar{\Lambda}}{\omega+1}\frac{1}{m_c}\xi(\omega), \quad \rho_3(\omega)=\rho_4(\omega)=0, \qquad \text{(E19)}
$$

$$
\xi_{A_3}(\omega) \;=\; \left[1+\frac{\bar{\Lambda}}{2}\left(\frac{\omega-1}{\omega+1}\frac{1}{m_c}+\frac{1}{m_b}\right)+\left(\frac{1}{m_c}+\frac{1}{m_b}\right)\rho_1(\omega)\right]\xi_+(\omega), \qquad \text{(E20)}
$$

$$
\rho_2(\omega) \;=\; \rho_1(\omega), \qquad \rho_3(\omega)=\rho_4(\omega)=0. \qquad \text{(E21)}
$$

Therefore all the expressions for $\rho_i(\omega)$ included in $\xi_i(\omega)$ are consistent from each other up to the first order in $1/m_Q$. Even though the first order corrections to the wave functions are present as given by Eqs. (27), their contributions vanish because of matrix structure after taking a trace over indices. From the above equations one can easily reproduce our results given by Eqs. (58) together with Eqs.(59).

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TABLE I: Corresponding values for k, s_ℓ, and P.

P	$(-)^{j+1}$	$(-)^{j}$	$(-)^{j}$	$(-)^{j+1}$
k	$-(j+1)$	$j+1$	$-j$	j
s_ℓ	$j+\frac{1}{2}$	$j+\frac{1}{2}$	$j-\frac{1}{2}$	$j-\frac{1}{2}$
Ψ_j^k	$\Psi_j^{-(j+1)}$	Ψ_j^{j+1}	Ψ_j^{-j}	Ψ_j^{j}

TABLE II: States classified by various quantum numbers.

j^P	0^-	1^-	0^+	1^+	1^+	2^+	1^-	2^-	2^-	2^+
k	-1	-1	1	1	-2	-2	2	2	-3	3
s_ℓ^P	$\frac{1}{2}^-$	$\frac{1}{2}^-$	$\frac{1}{2}^+$	$\frac{1}{2}^+$	$\frac{3}{2}^+$	$\frac{3}{2}^+$	$\frac{3}{2}^-$	$\frac{3}{2}^-$	$\frac{5}{2}^-$	$\frac{5}{2}^+$
Ψ_j^k	Ψ_0^{-1}	Ψ_1^{-1}	Ψ_0^1	Ψ_1^1	Ψ_1^{-2}	Ψ_2^{-2}	Ψ_1^2	Ψ_2^2	Ψ_2^{-3}	Ψ_2^3